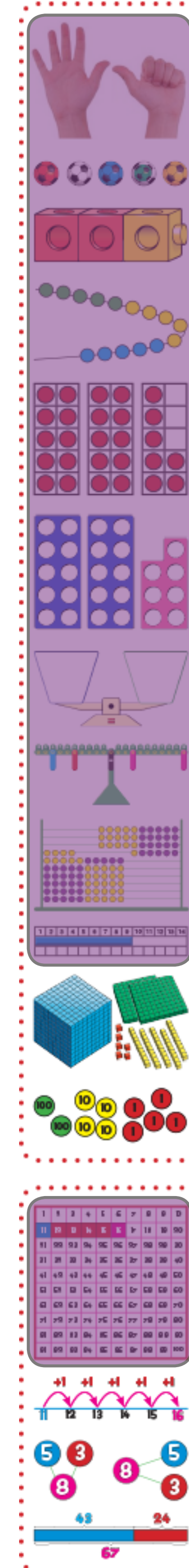


6

100	10	1
1 ¹²	3 ¹	2
	5	6
	7	6

Tens		Ones
		
		
		
		
		

Tens	Ones

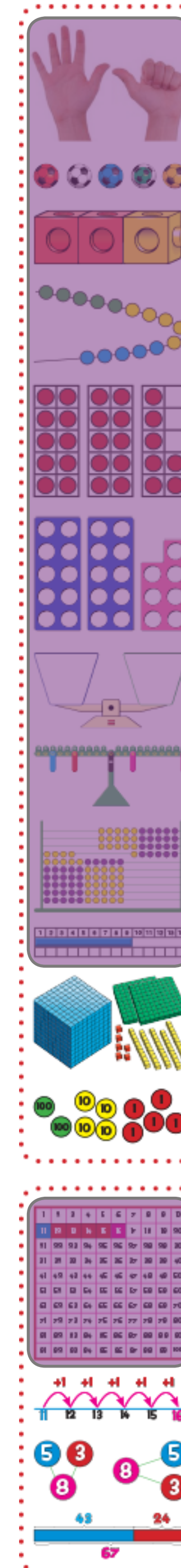
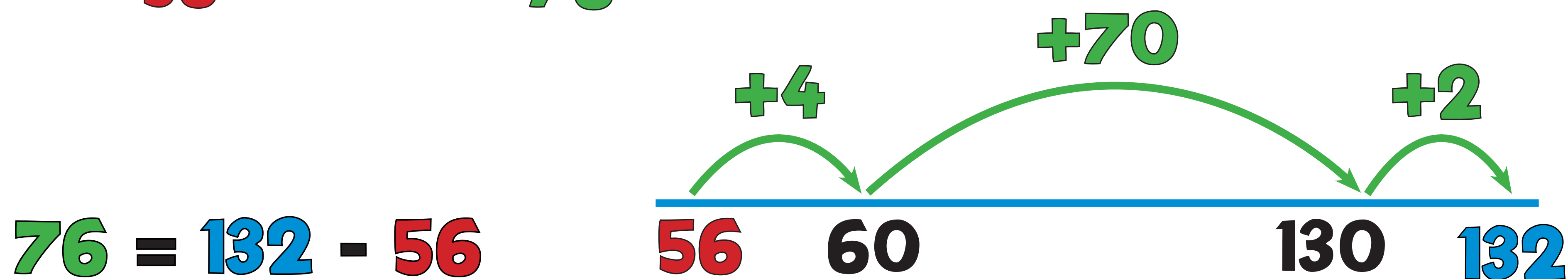
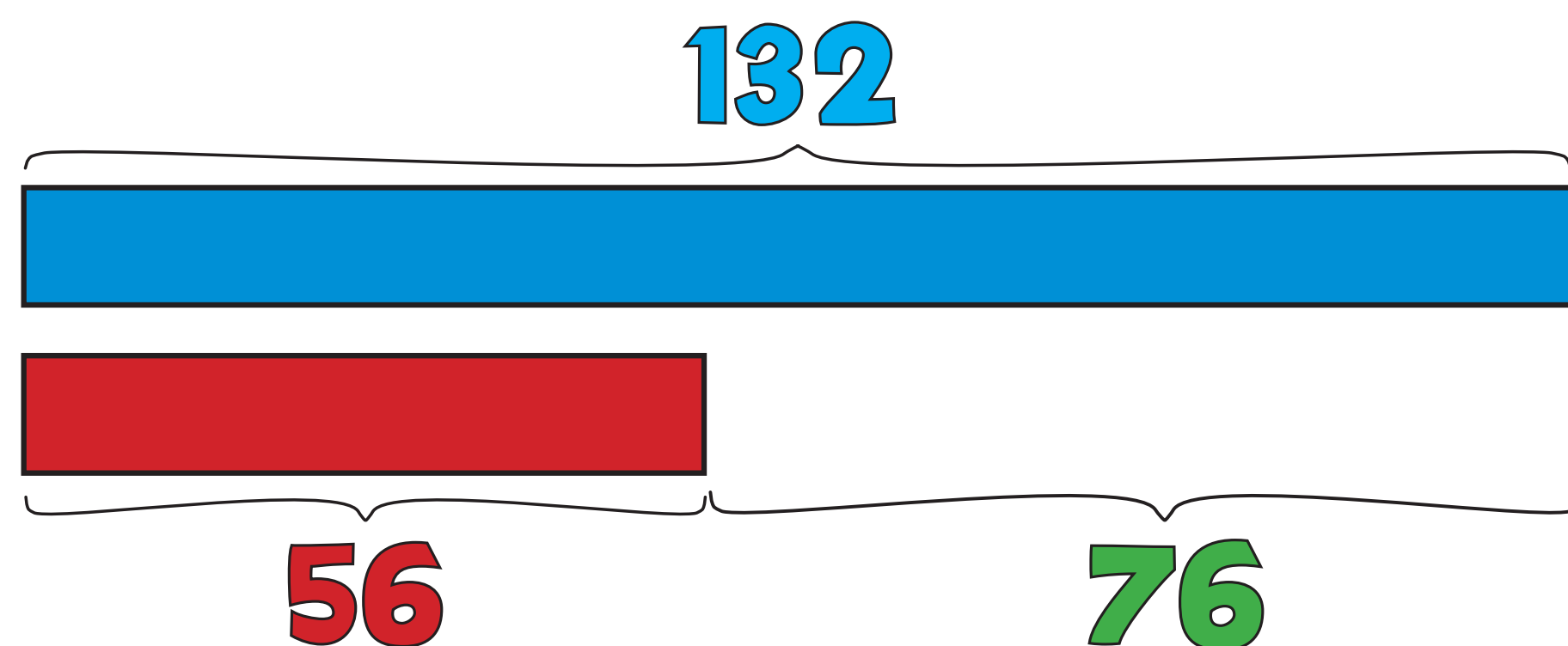
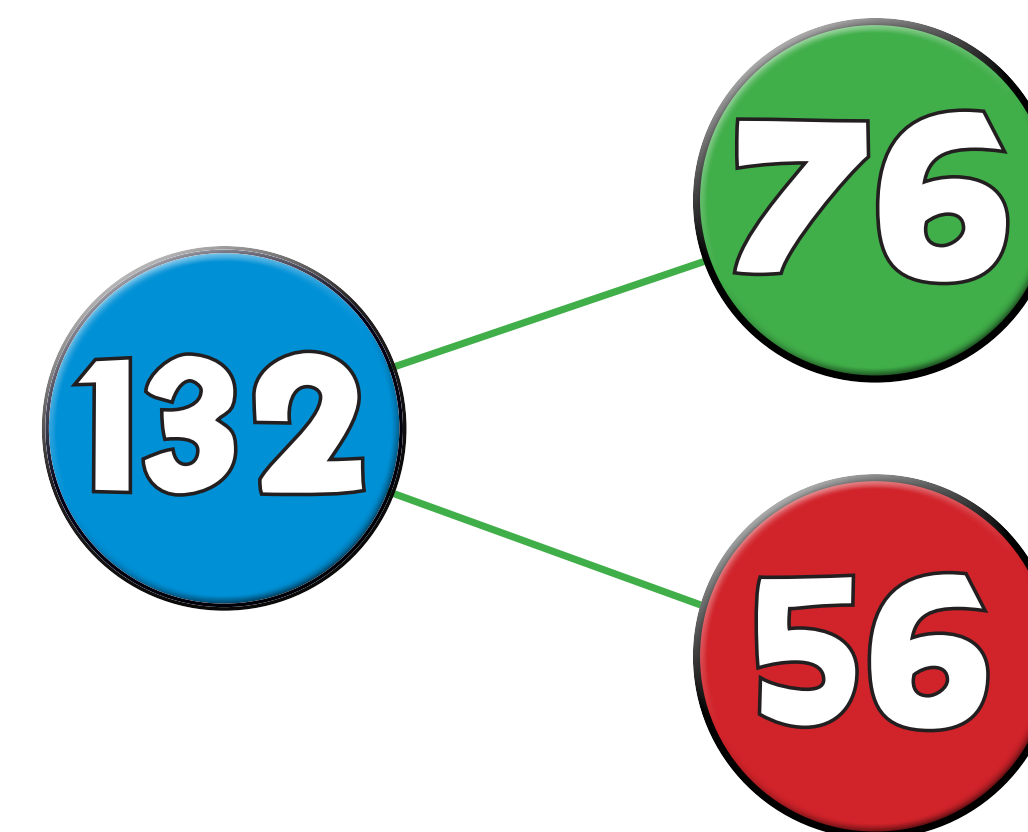
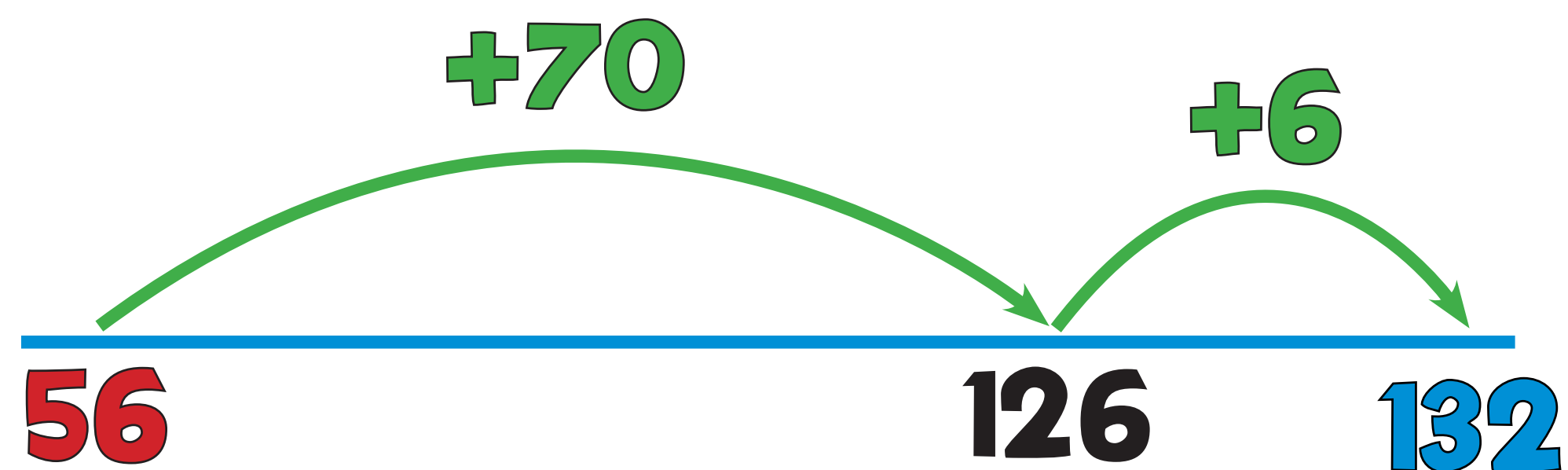


CPA
↔
reasoning

132 - 56 = 76

CPA
↔
reasoning

132 - 56 = 76



$$736 - 462 = 274$$

1

100 10 1

$$\begin{array}{r} 736 \\ - 462 \\ \hline \end{array}$$

Hundreds Tens Ones

2

100 10 1

$$\begin{array}{r} 6736 \\ - 462 \\ \hline \end{array}$$

Hundreds Tens Ones

3

100 10 1

$$\begin{array}{r} 6736 \\ - 462 \\ \hline 4 \end{array}$$

Hundreds Tens Ones

4

100 10 1

$$\begin{array}{r} 6736 \\ - 462 \\ \hline 74 \end{array}$$

Hundreds Tens Ones

5

100 10 1

$$\begin{array}{r} 6736 \\ - 462 \\ \hline 74 \end{array}$$

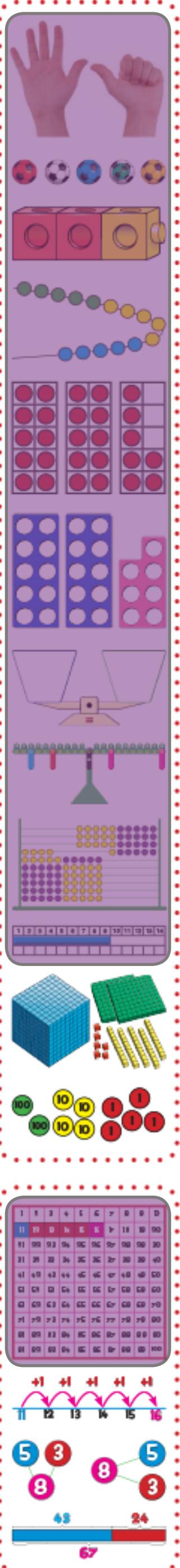
Hundreds Tens Ones

6

100 10 1

$$\begin{array}{r} 6736 \\ - 462 \\ \hline 274 \end{array}$$

Hundreds Tens Ones



$$723 - 356 = 367$$

1

100 10 1

$$\begin{array}{r} 723 \\ - 356 \\ \hline \end{array}$$

Hundreds Tens Ones

2

100 10 1

$$\begin{array}{r} 723 \\ - 356 \\ \hline \end{array}$$

Hundreds Tens Ones

3

100 10 1

$$\begin{array}{r} 723 \\ - 356 \\ \hline \end{array}$$

Hundreds Tens Ones

4

100 10 1

$$\begin{array}{r} 611 \\ 723 \\ - 356 \\ \hline 7 \end{array}$$

Hundreds Tens Ones

5

100 10 1

$$\begin{array}{r} 611 \\ 723 \\ - 356 \\ \hline 67 \end{array}$$

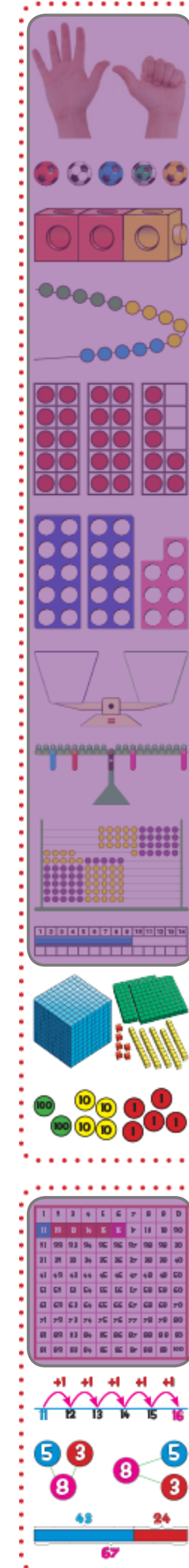
Hundreds Tens Ones

6

100 10 1

$$\begin{array}{r} 611 \\ 723 \\ - 356 \\ \hline 67 \end{array}$$

Hundreds Tens Ones



723 - 356 = 367

7

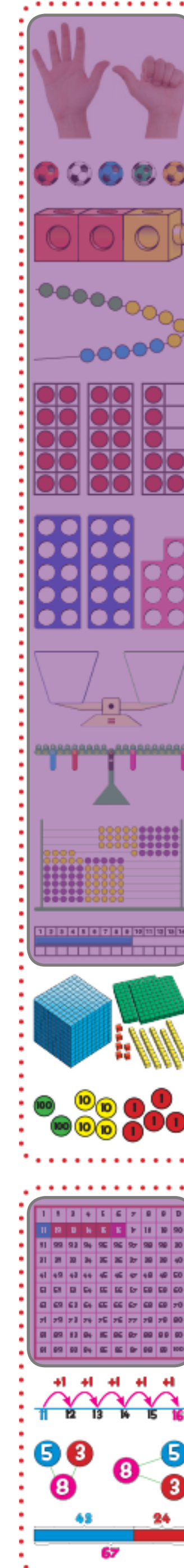
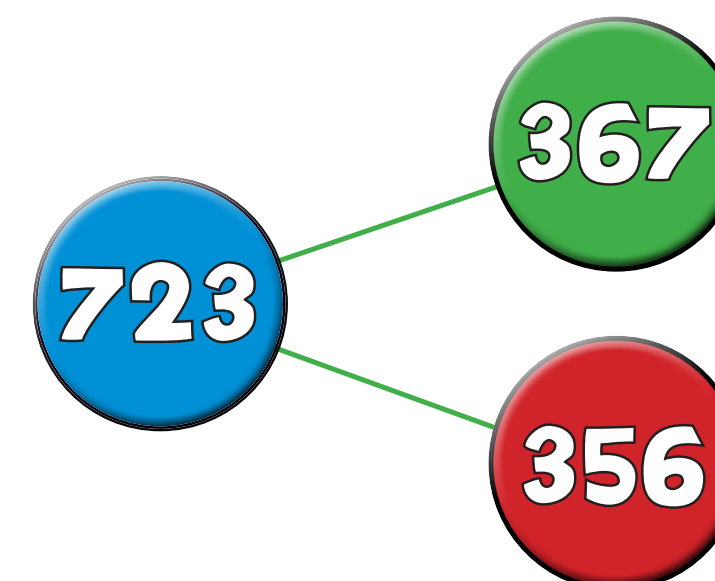
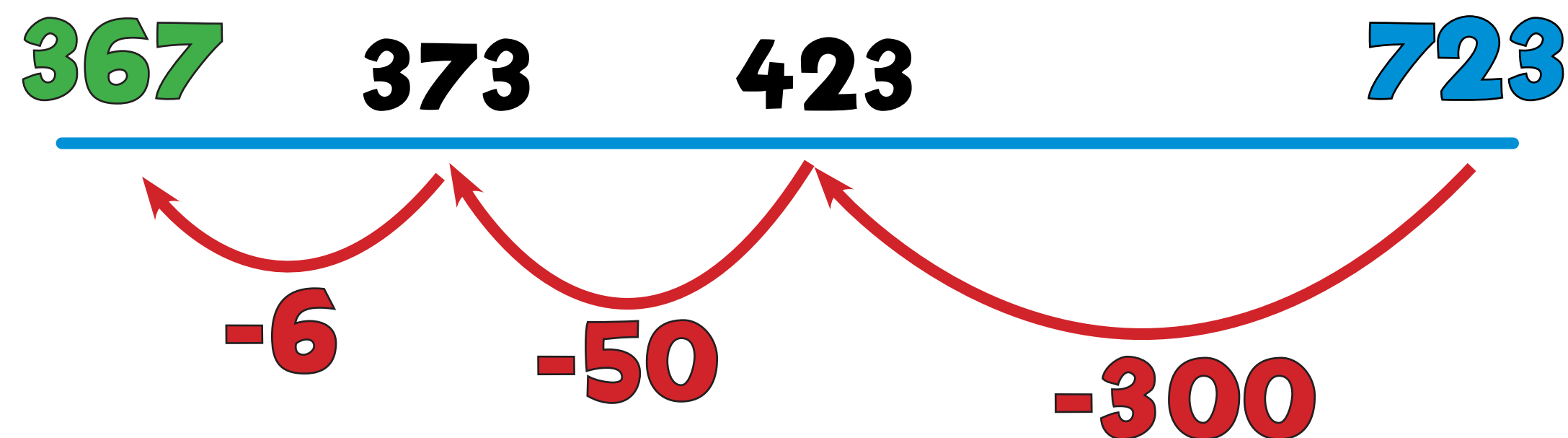
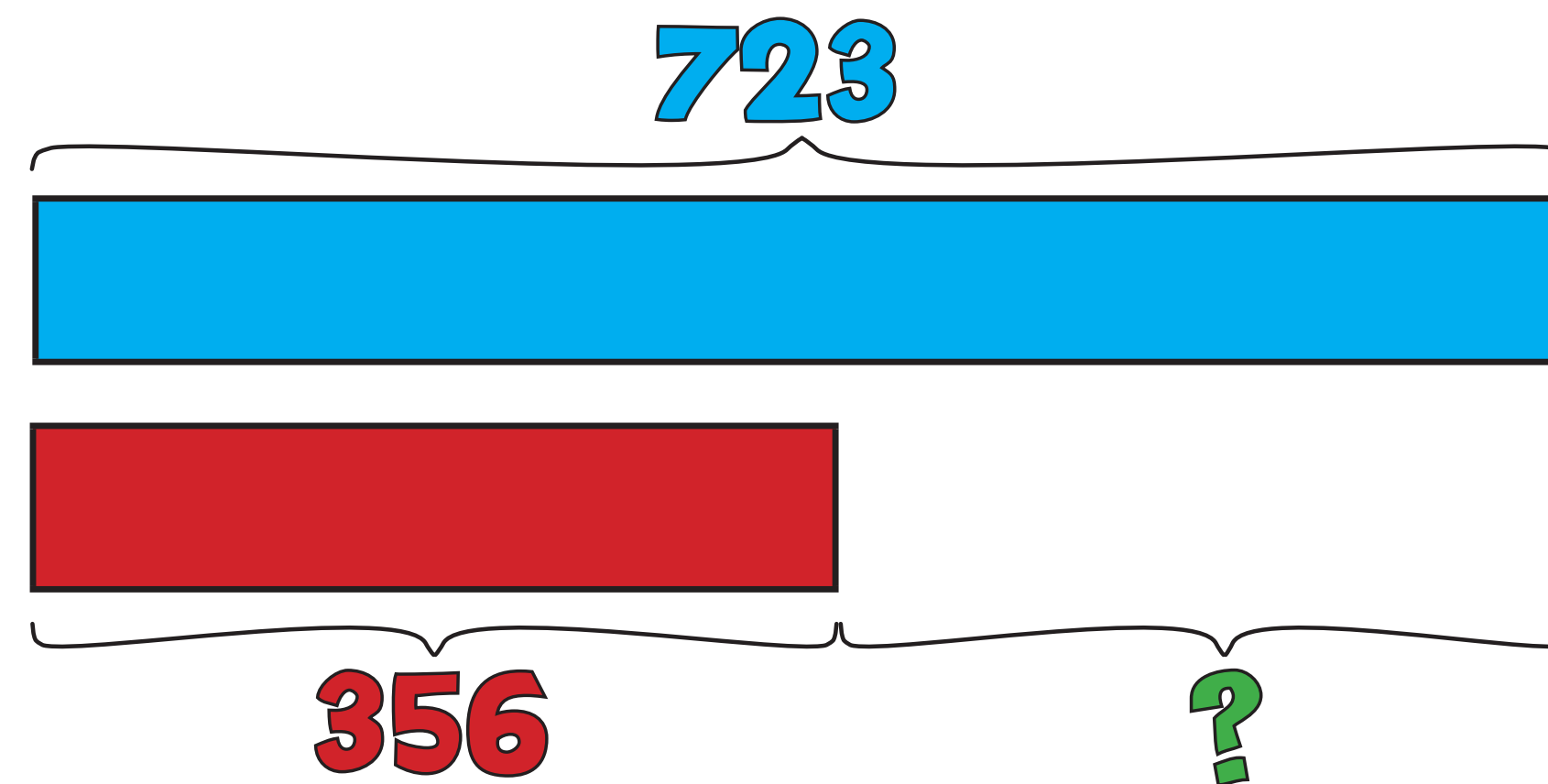
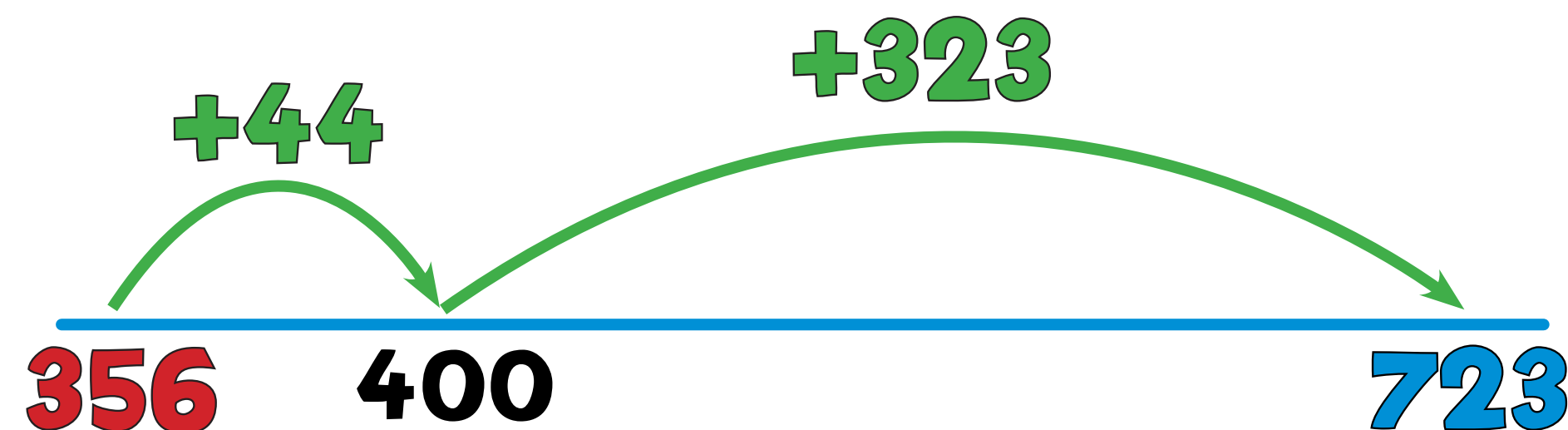
$$\begin{array}{r} 100 \quad 10 \quad 1 \\ 6 \quad 11 \quad 1 \\ 723 \\ - 356 \\ \hline 67 \end{array}$$

Hundreds Tens Ones

8

$$\begin{array}{r} 100 \quad 10 \quad 1 \\ 6 \quad 11 \quad 1 \\ 723 \\ - 356 \\ \hline 367 \end{array}$$

Hundreds Tens Ones



$$1375 - 538 = 837$$

1

Thousands Hundreds Tens Ones

1000 100 10 1

1375
- 538

2

Thousands Hundreds Tens Ones

1000 100 10 1

1375
- 538

3

Thousands Hundreds Tens Ones

1000 100 10 1

1375
- 538

4

Thousands Hundreds Tens Ones

1000 100 10 1

1375
- 538

5

Thousands Hundreds Tens Ones

1000 100 10 1

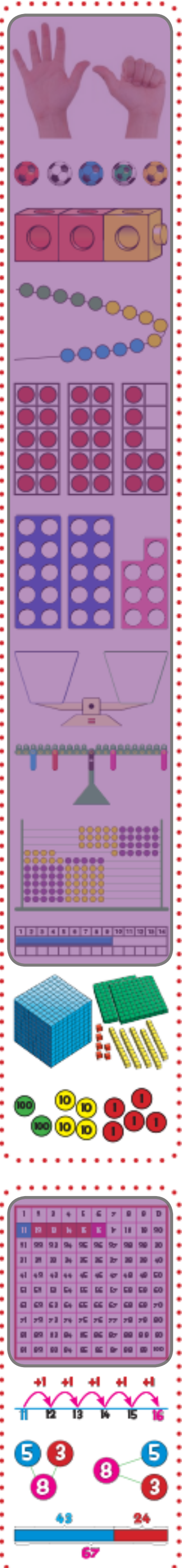
1375
- 538

6

Thousands Hundreds Tens Ones

1000 100 10 1

1375
- 538



$$1375 - 538 = 837$$

7

1000 100 10 1

$$\begin{array}{r} 1375 \\ - 538 \\ \hline 37 \end{array}$$

Thousands Hundreds Tens Ones

8

1000 100 10 1

$$\begin{array}{r} 1375 \\ - 538 \\ \hline 37 \end{array}$$

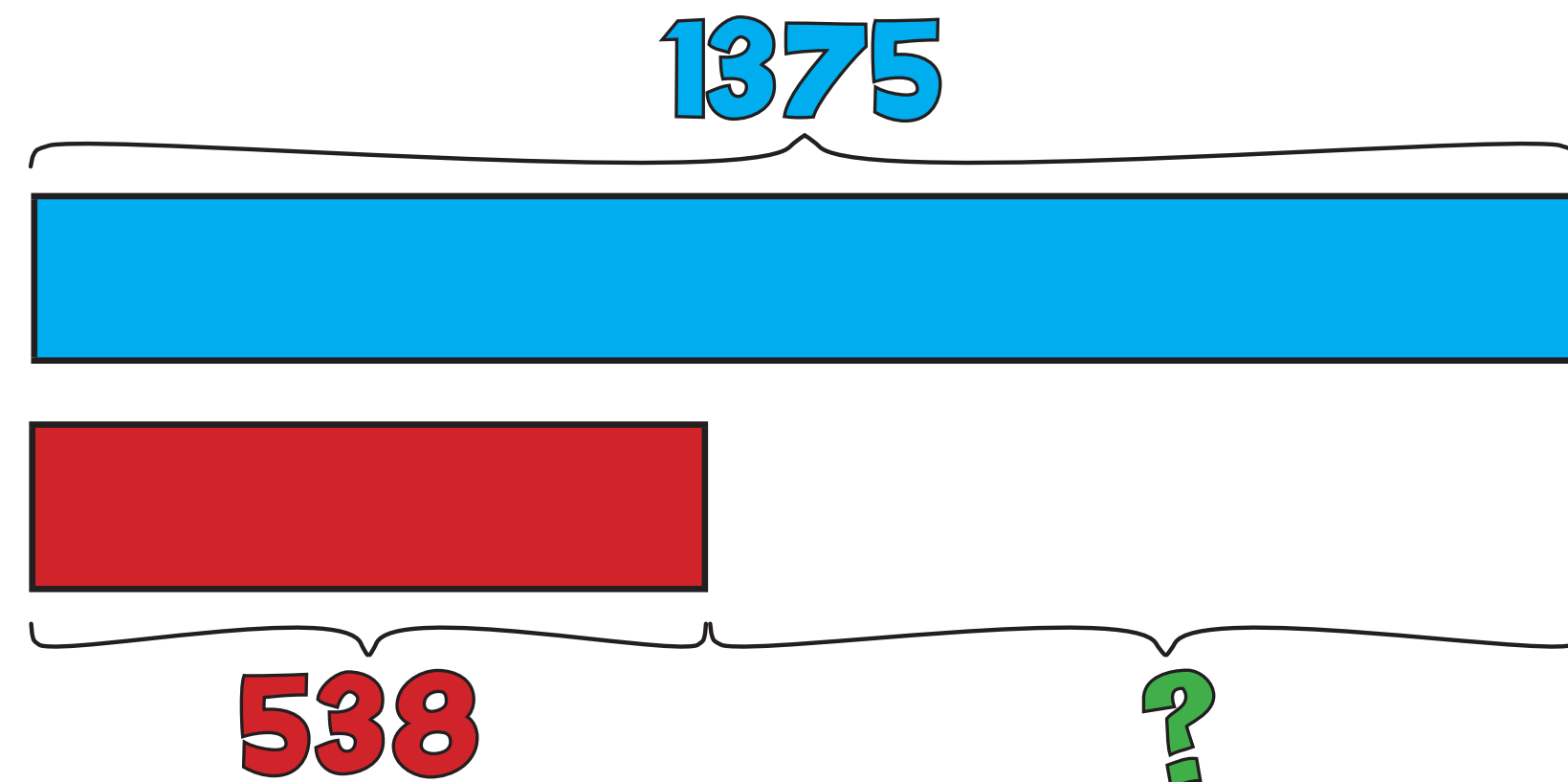
Thousands Hundreds Tens Ones

9

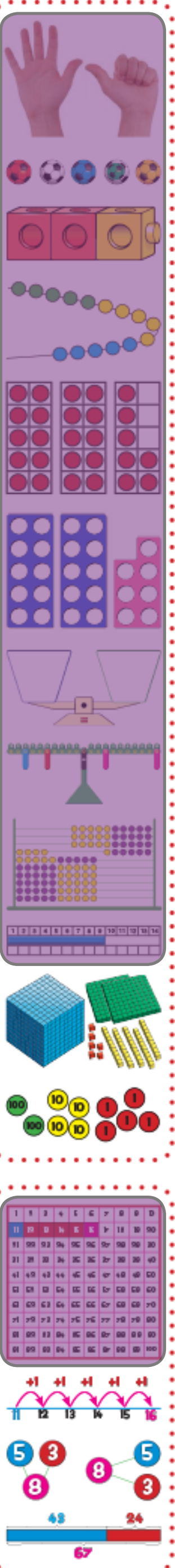
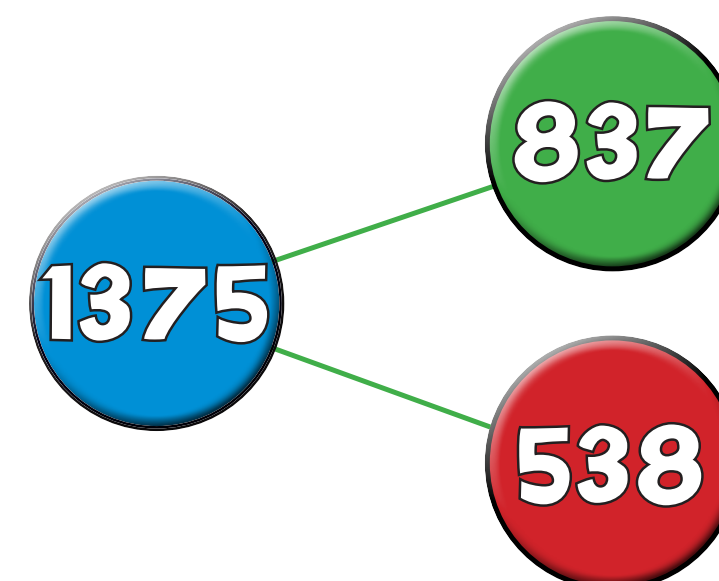
1000 100 10 1

$$\begin{array}{r} 1375 \\ - 538 \\ \hline 837 \end{array}$$

Thousands Hundreds Tens Ones



It's a
'take away'
image!



$$5042 - 1776 = 3266$$

7

Thousands Hundreds Tens Ones

1000 100 10 1

$\begin{array}{r} 5042 \\ - 1776 \\ \hline 6 \end{array}$

6

8

Thousands Hundreds Tens Ones

1000 100 10 1

$\begin{array}{r} 5042 \\ - 1776 \\ \hline 66 \end{array}$

76

9

Thousands Hundreds Tens Ones

1000 100 10 1

$\begin{array}{r} 5042 \\ - 1776 \\ \hline 66 \end{array}$

76

10

Thousands Hundreds Tens Ones

1000 100 10 1

$\begin{array}{r} 5042 \\ - 1776 \\ \hline 266 \end{array}$

776

11

Thousands Hundreds Tens Ones

1000 100 10 1

$\begin{array}{r} 5042 \\ - 1776 \\ \hline 266 \end{array}$

776

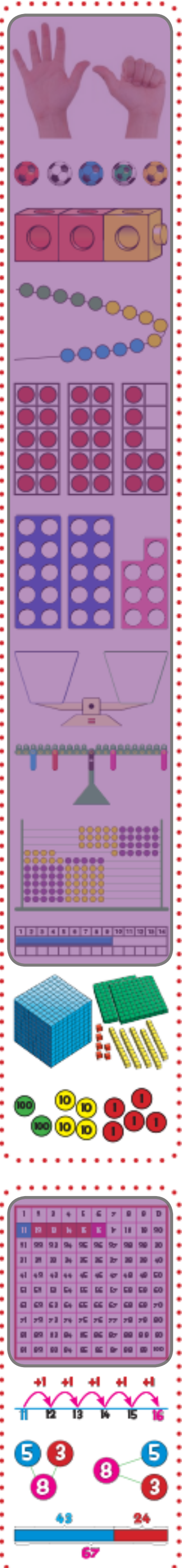
12

Thousands Hundreds Tens Ones

1000 100 10 1

$\begin{array}{r} 5042 \\ - 1776 \\ \hline 3266 \end{array}$

1776



$$13.4 - 8.7 = 4.7$$

1

10 1 $\frac{1}{10}$

13.4
- 8.7

Tens Ones Tenths

10

1
1
1

0.1
0.1
0.1
0.1

2

10 1 $\frac{1}{10}$

13.4
- 8.7

Tens Ones Tenths

10

1
1

0.1
0.1
0.1
0.1
0.1
0.1

1

3

10 1 $\frac{1}{10}$

13.4
- 8.7

Tens Ones Tenths

10

1
1

0.1
0.1
0.1
0.1
0.1
0.1
0.1
0.1

4

10 1 $\frac{1}{10}$

13.4
- 8.7

Tens Ones Tenths

10

1

0.1
0.1
0.1
0.1
0.1
0.1
0.1
0.1

5

10 1 $\frac{1}{10}$

13.4
- 8.7

Tens Ones Tenths

10

1
1

0.1
0.1
0.1
0.1
0.1
0.1

6

10 1 $\frac{1}{10}$

13.4
- 8.7

4.7

Ones Tenths

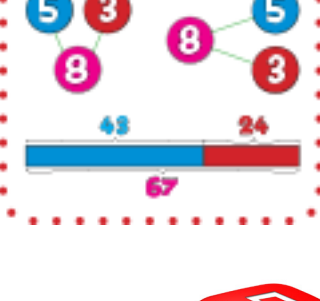
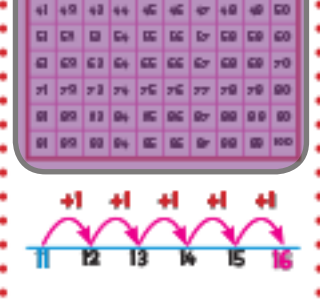
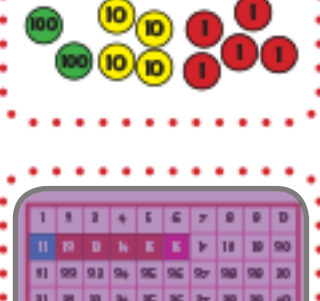
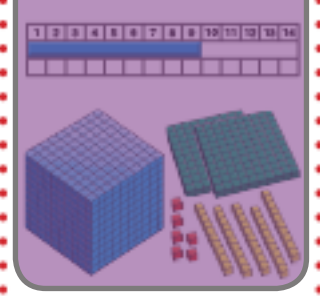
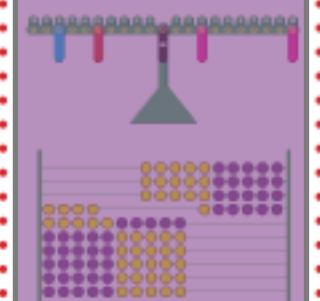
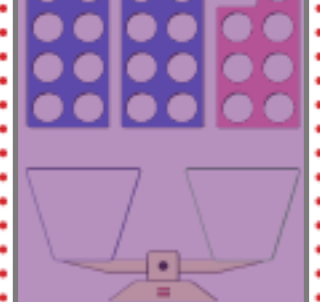
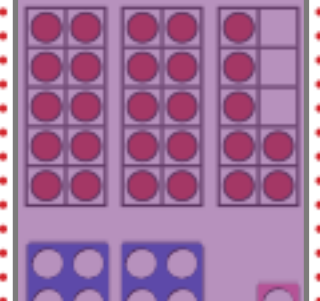
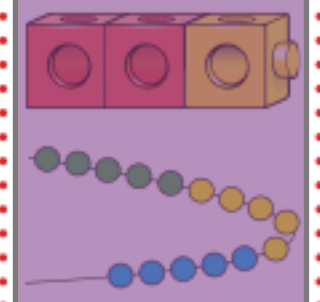
Ones Tenths

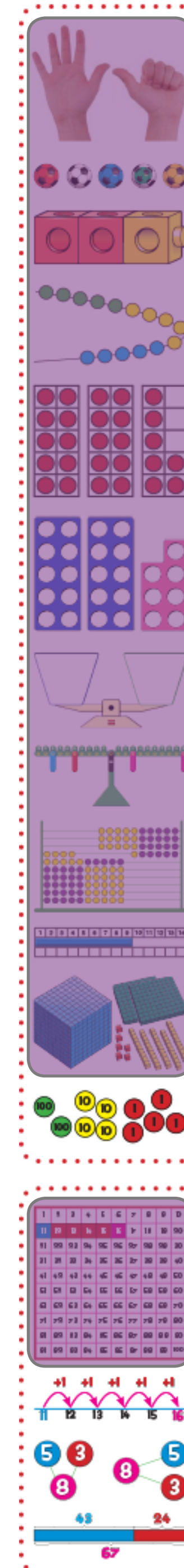
1
1
1
1

0.1
0.1
0.1
0.1
0.1
0.1

1
1
1
1

0.1
0.1
0.1
0.1
0.1
0.1



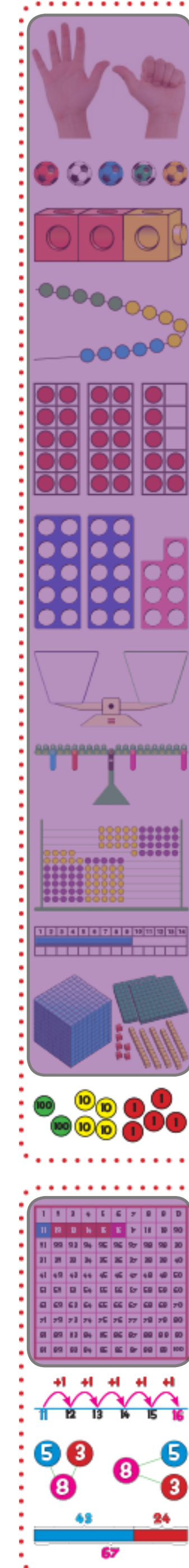
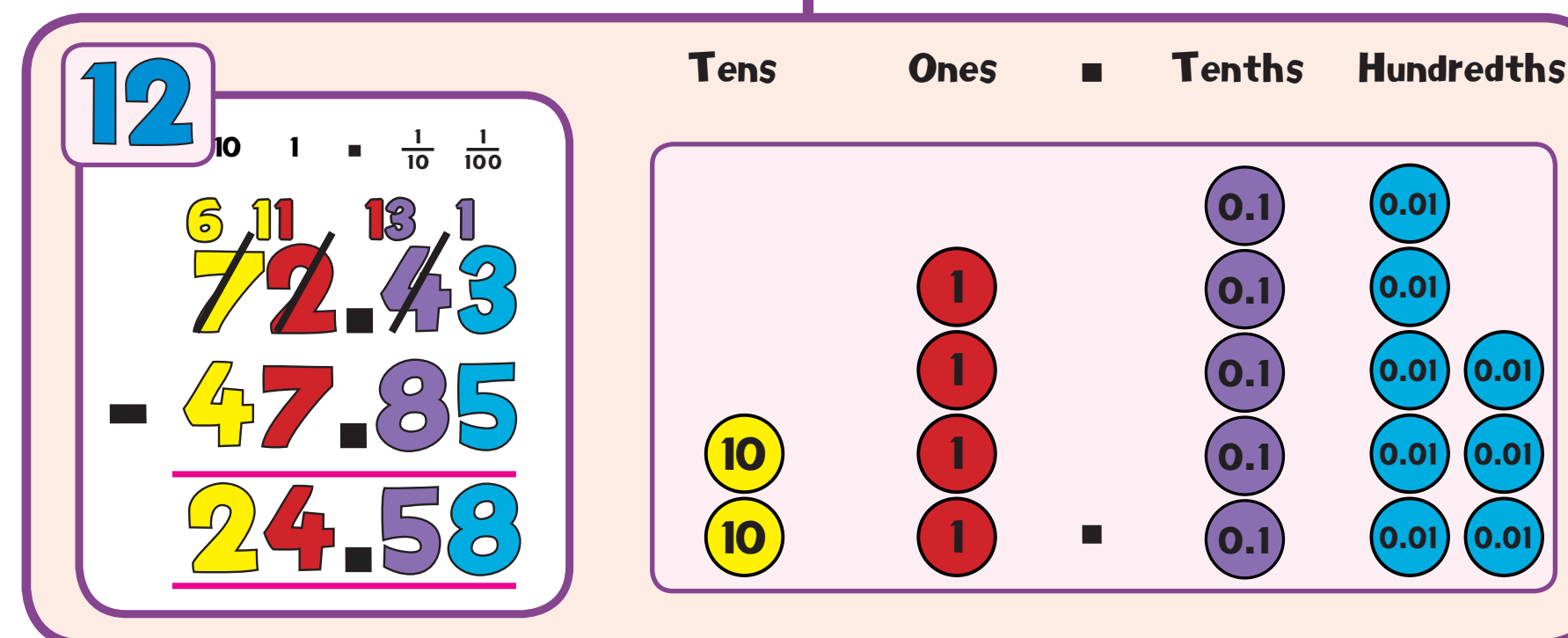
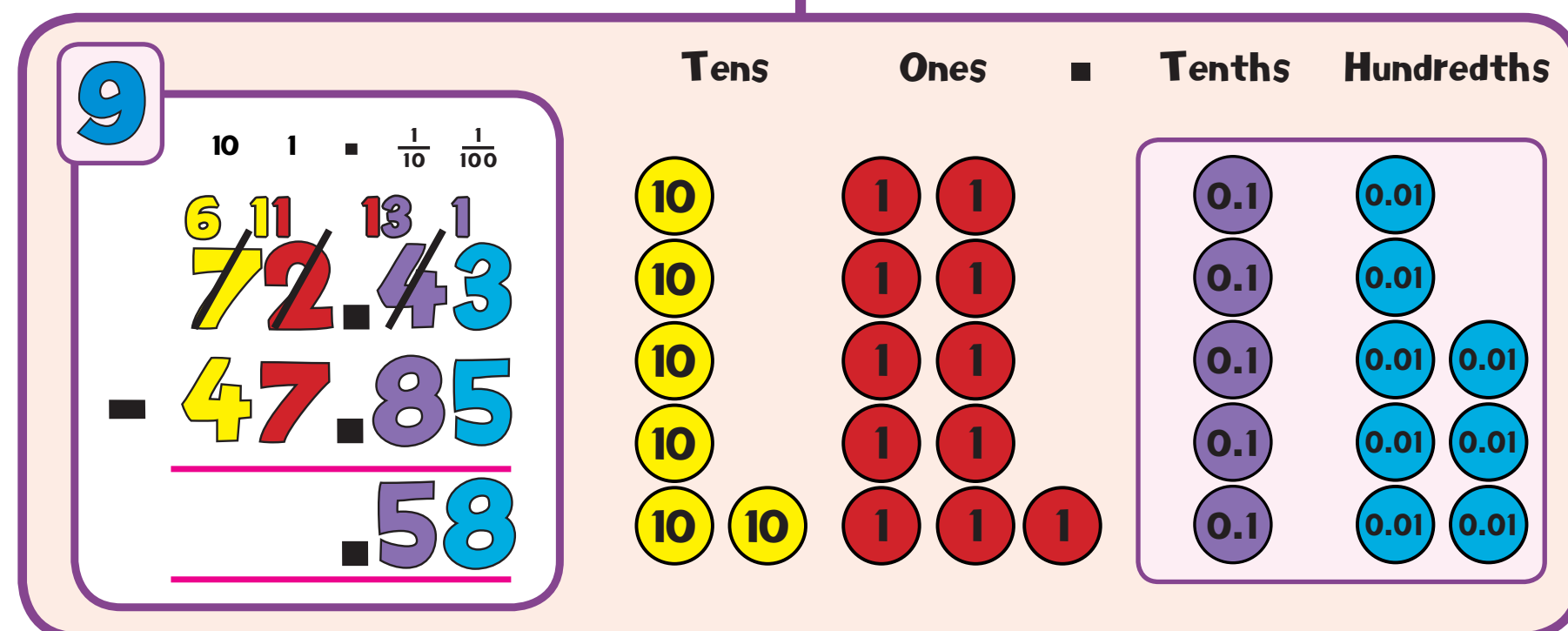
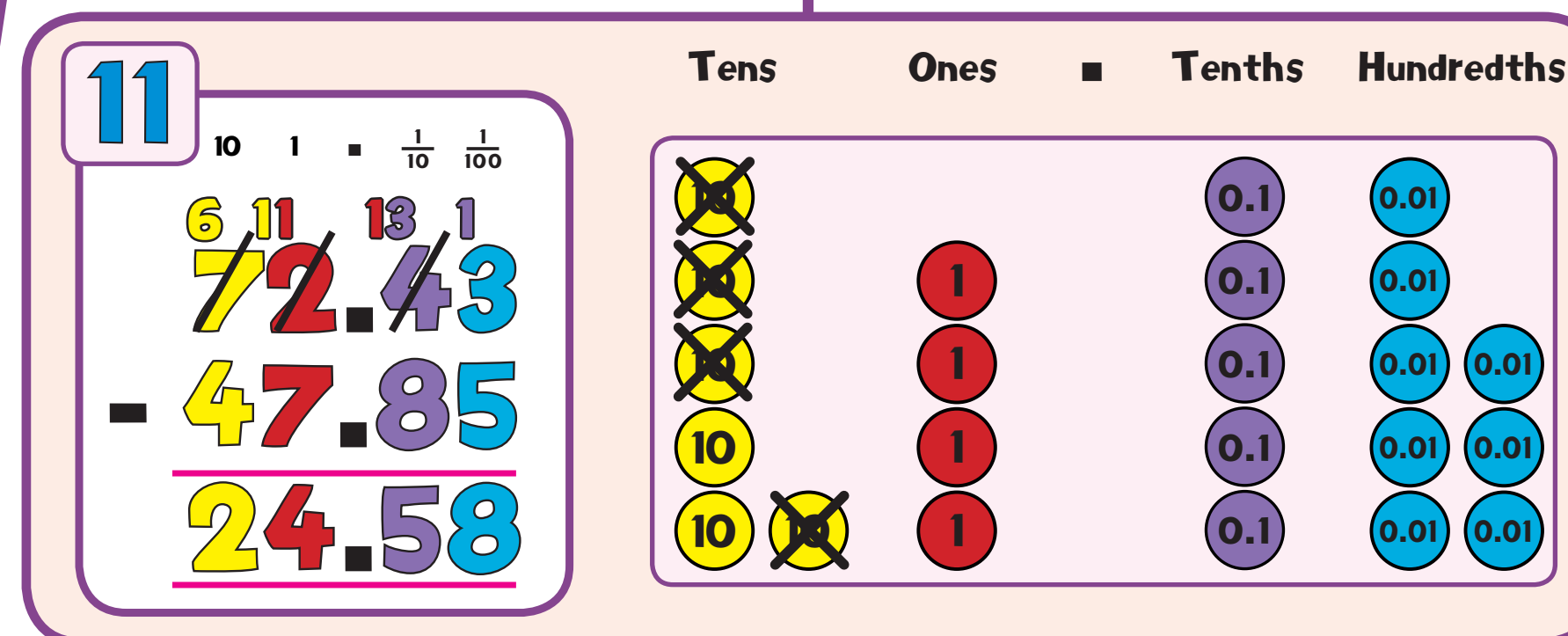
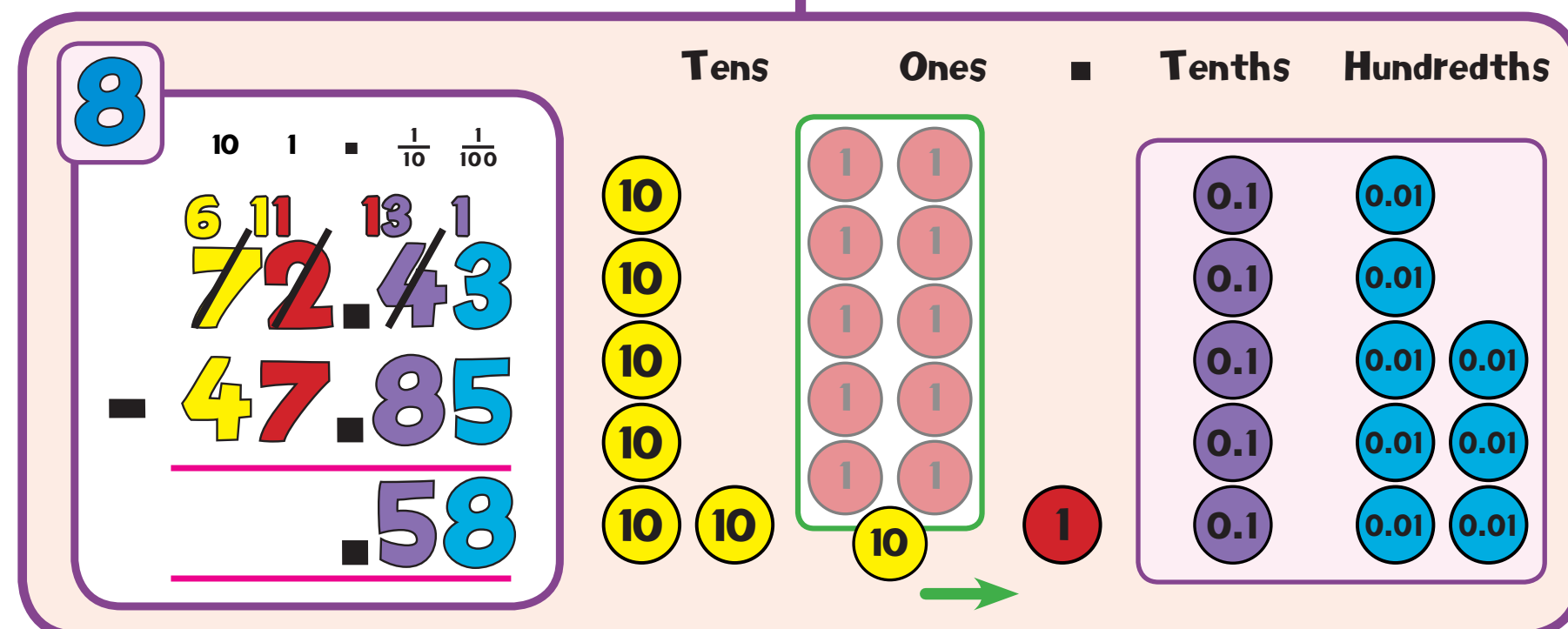
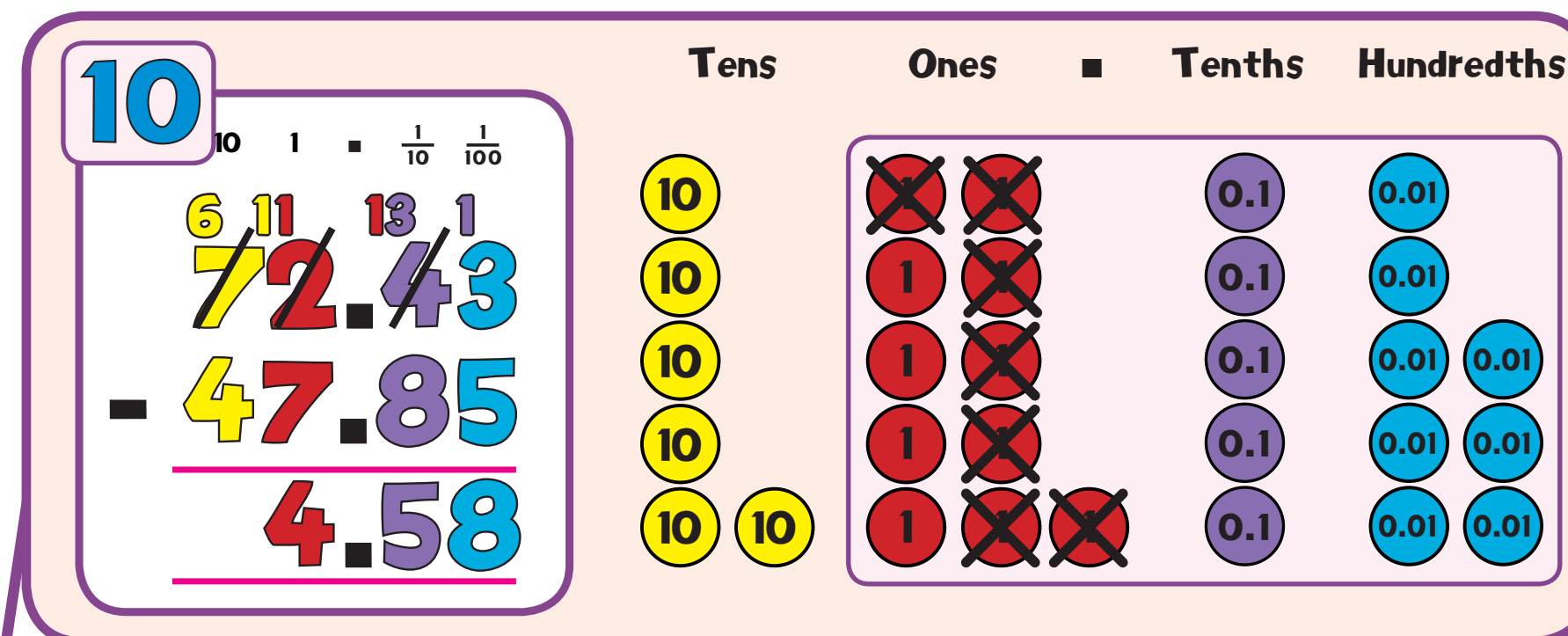
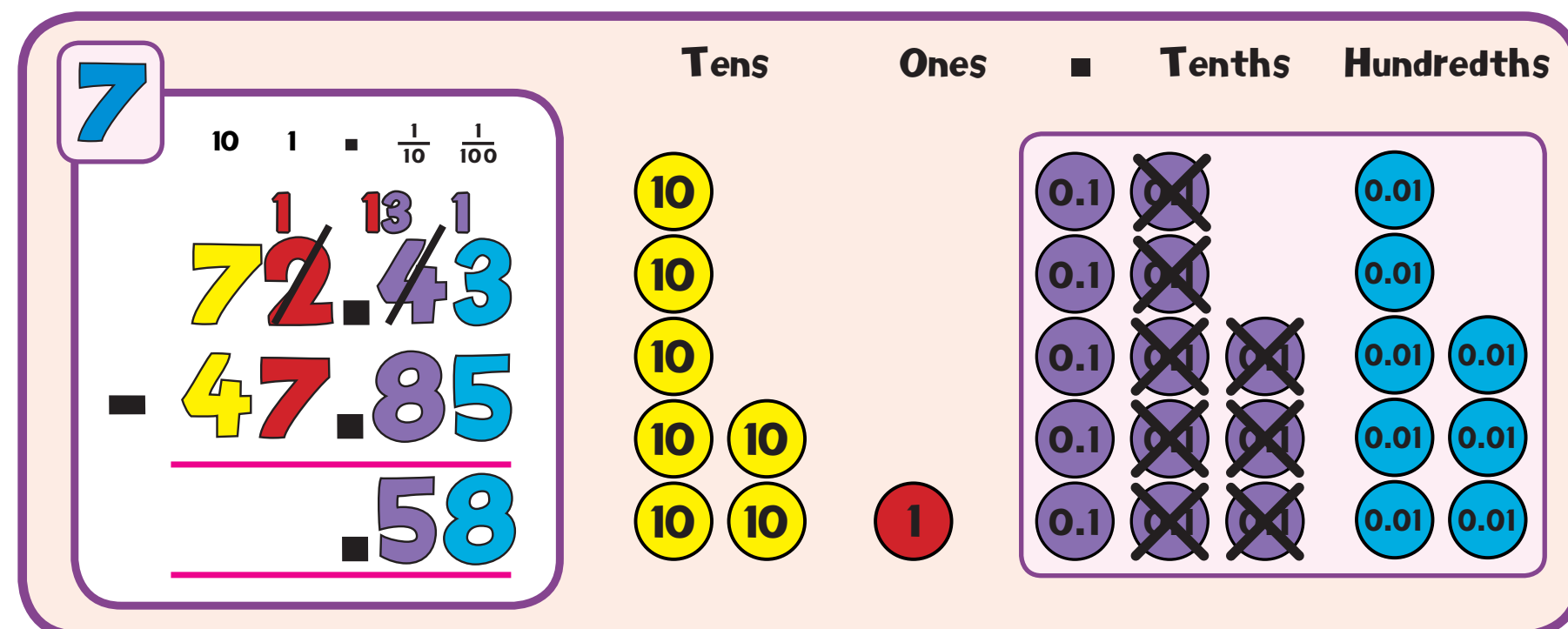


6

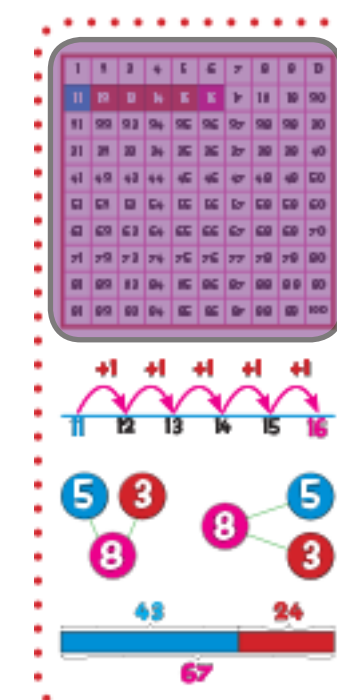
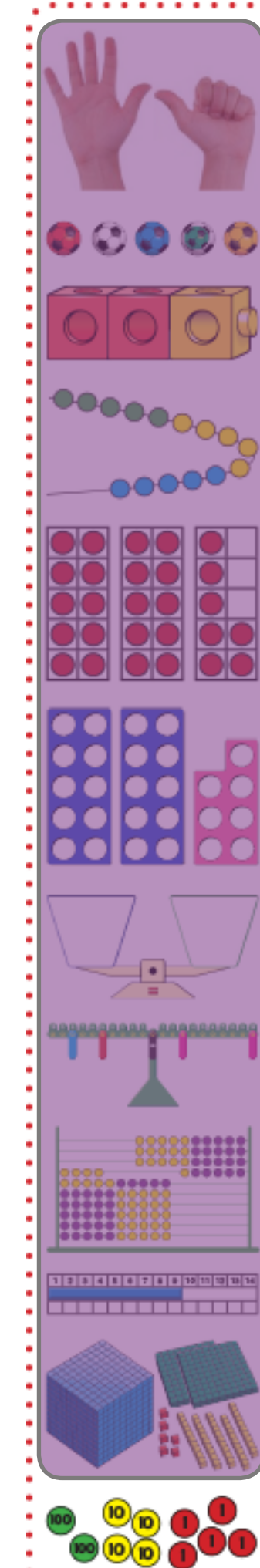
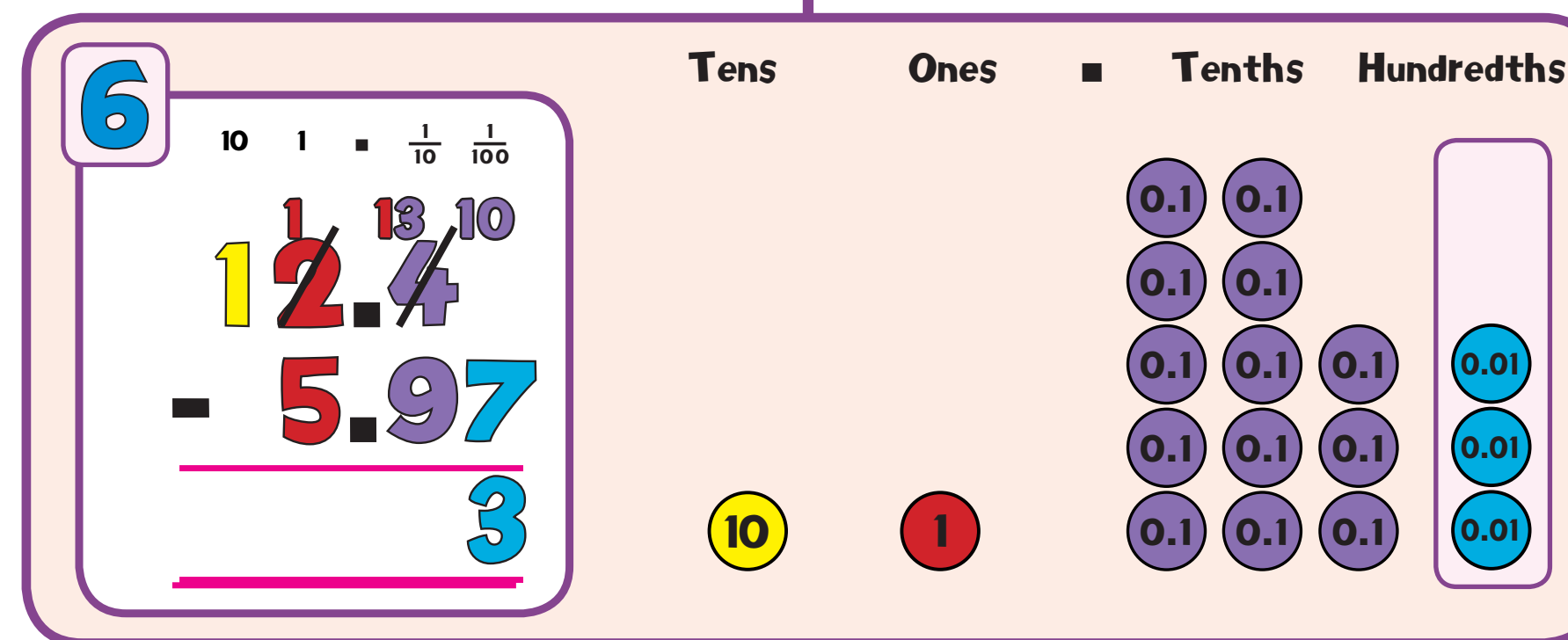
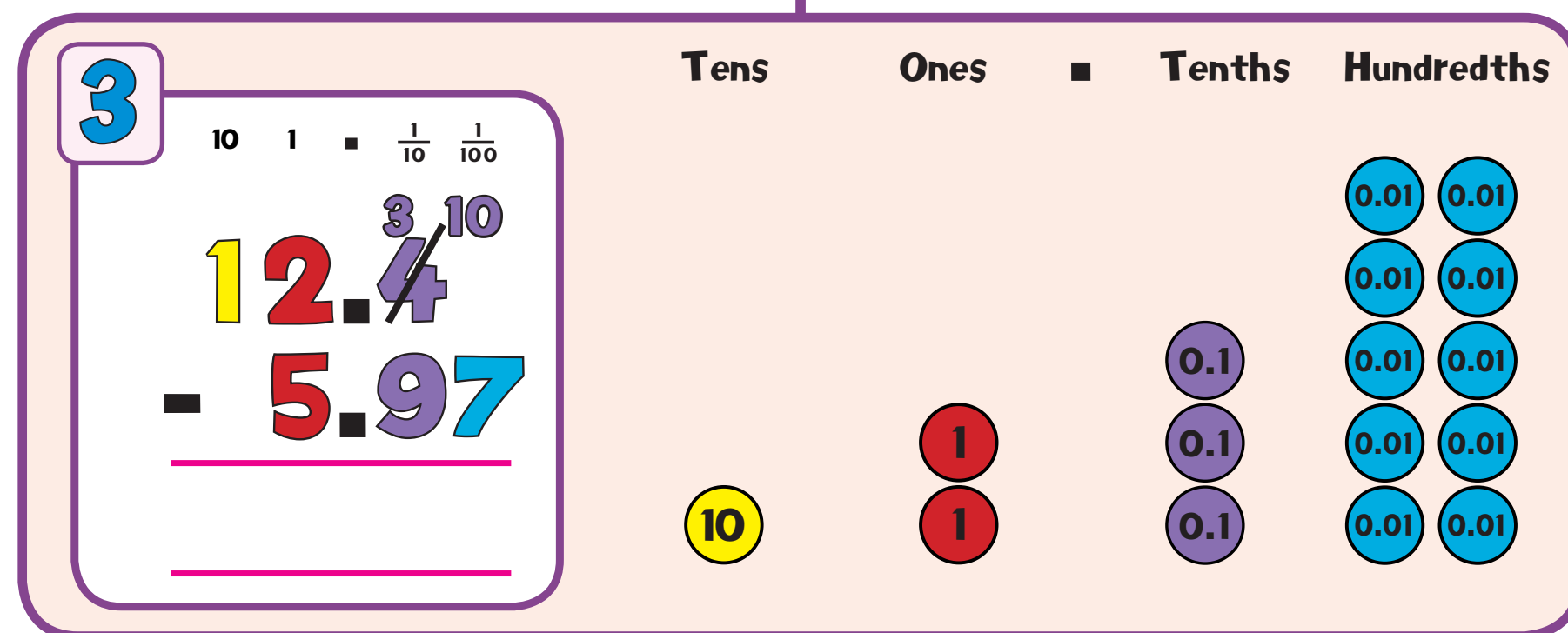
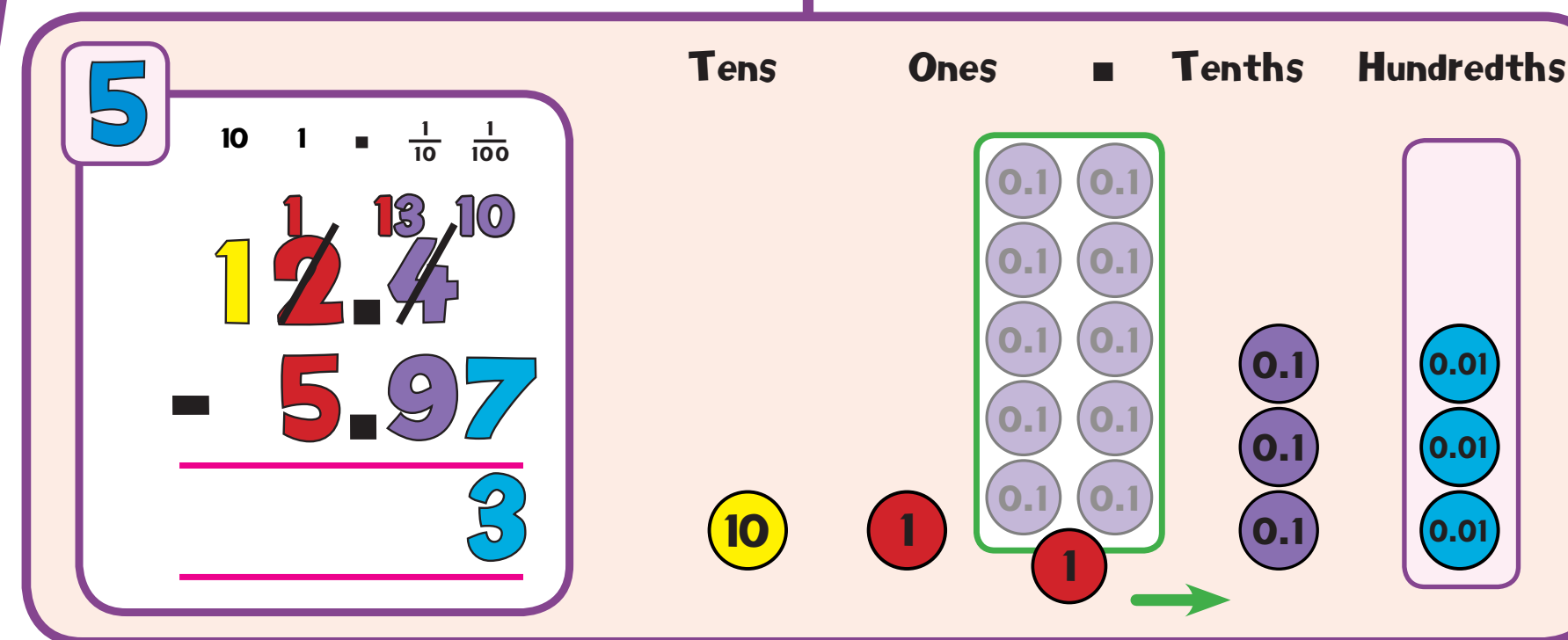
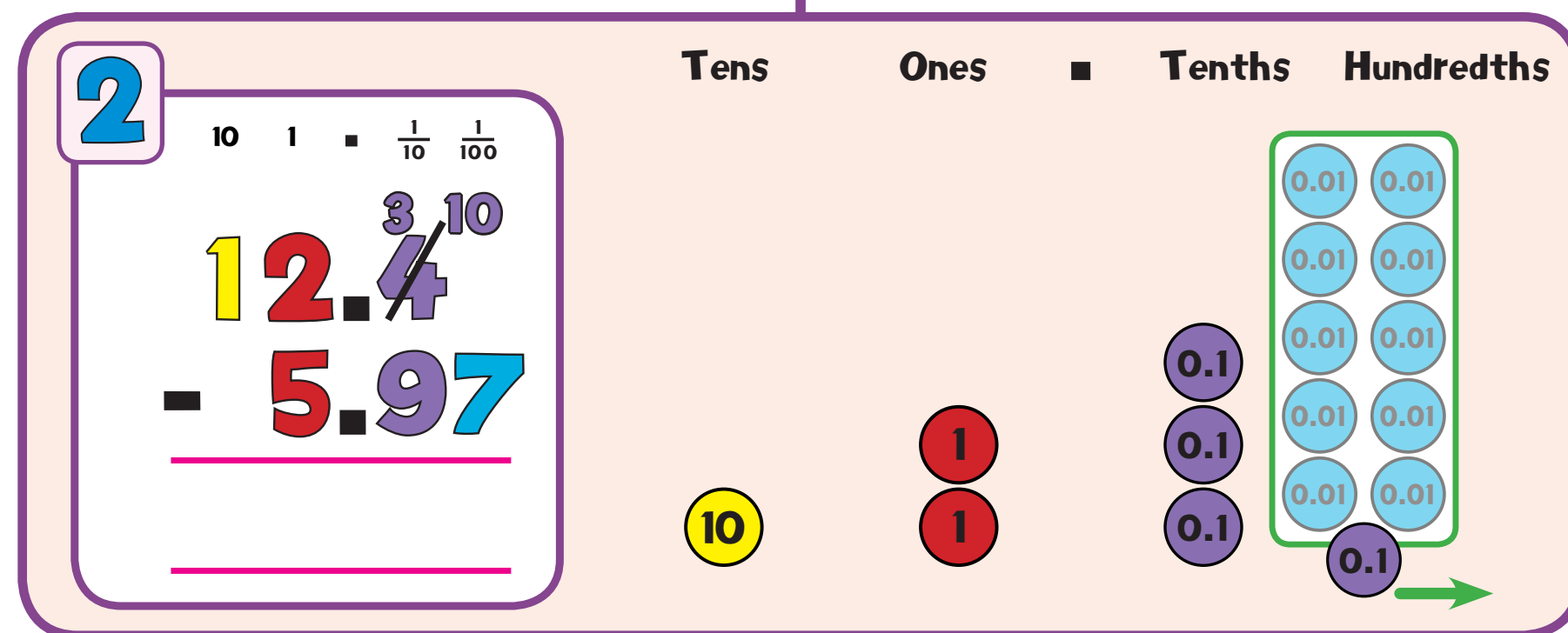
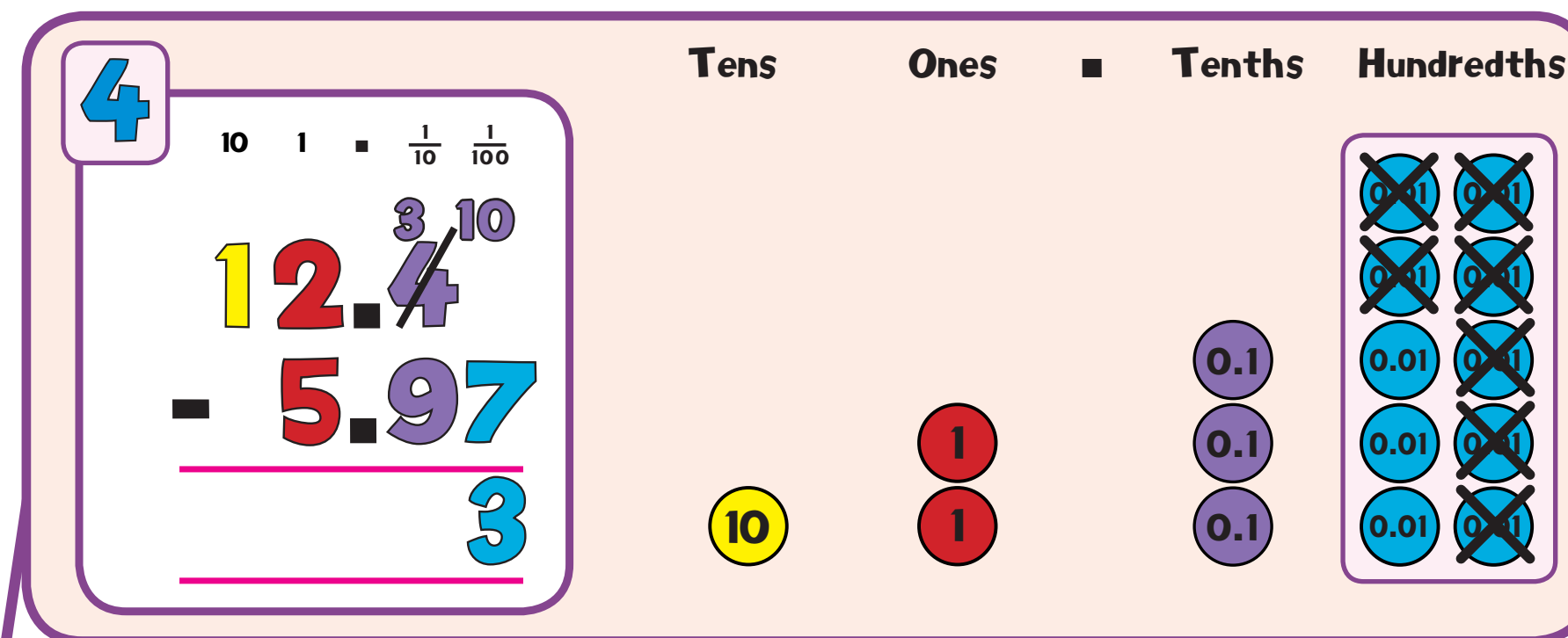
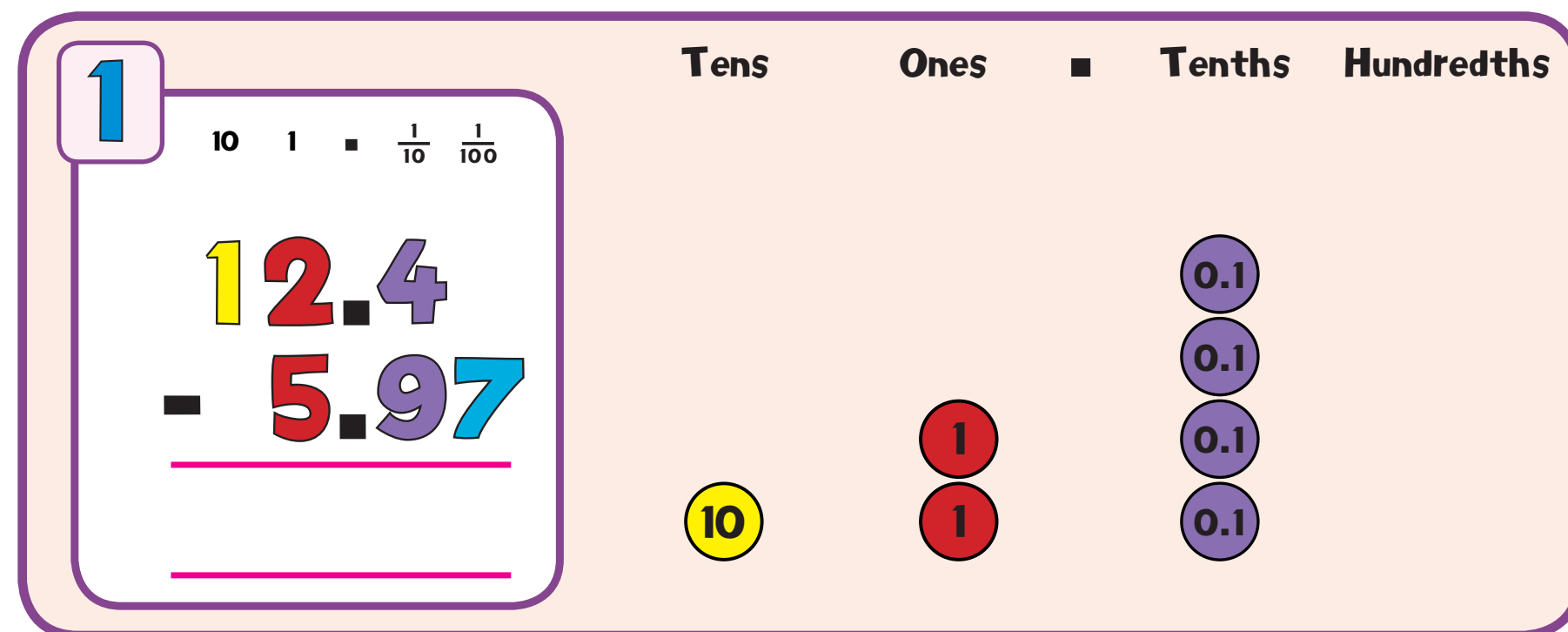
10	1	■	$\frac{1}{10}$	$\frac{1}{100}$
7	2	.	4	3
4	7	.	8	5
				8

Tens	Ones	■	Tenths	Hundredths
10 10 10 10 10 10 10	1		0.1 0.1 0.1 0.1 0.1 0.1 0.1	0.01 0.01 0.01 0.01 0.01 0.01 0.01

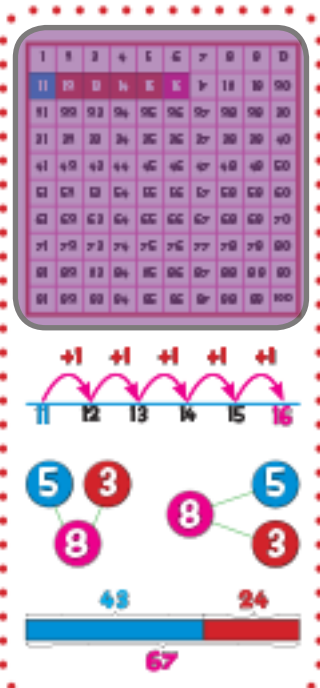
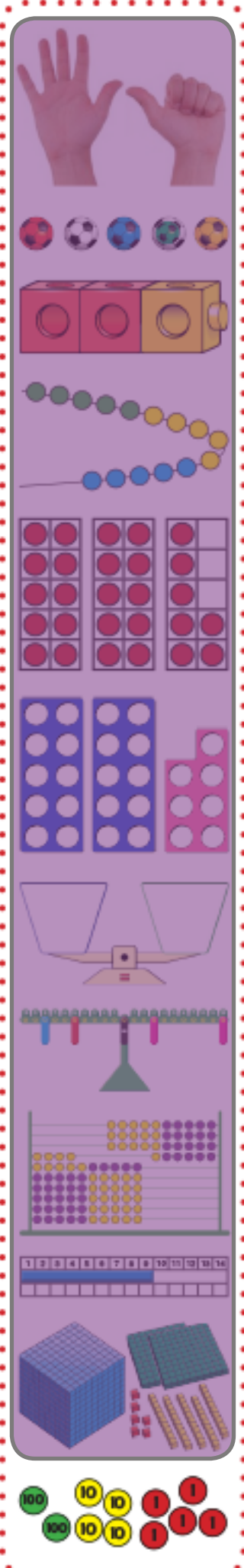
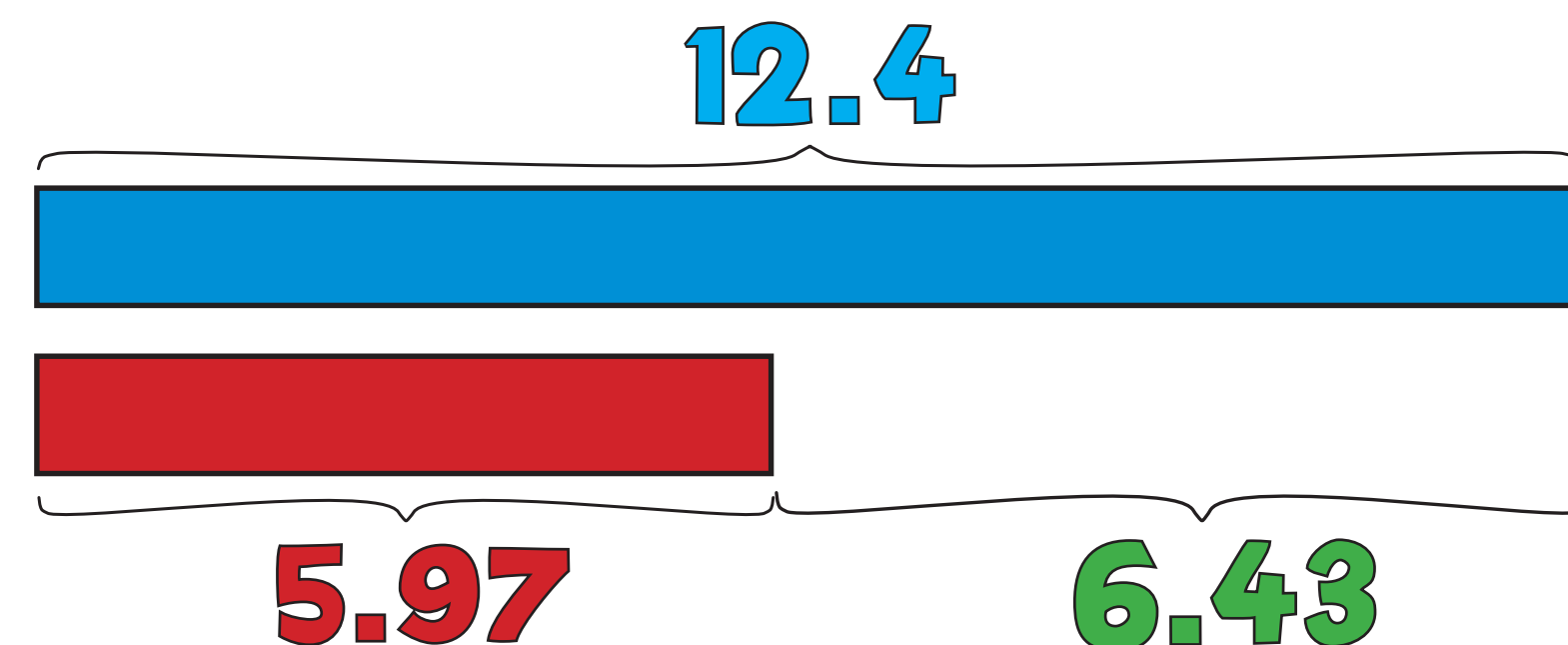
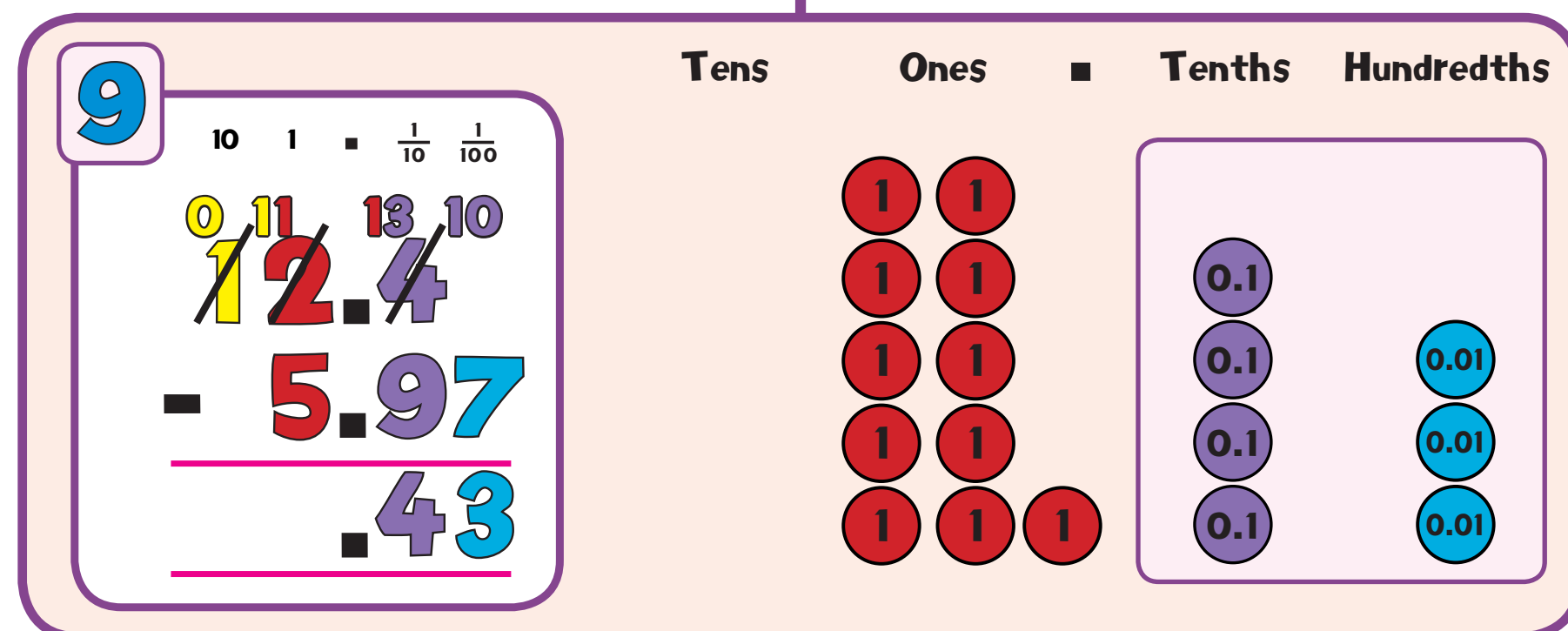
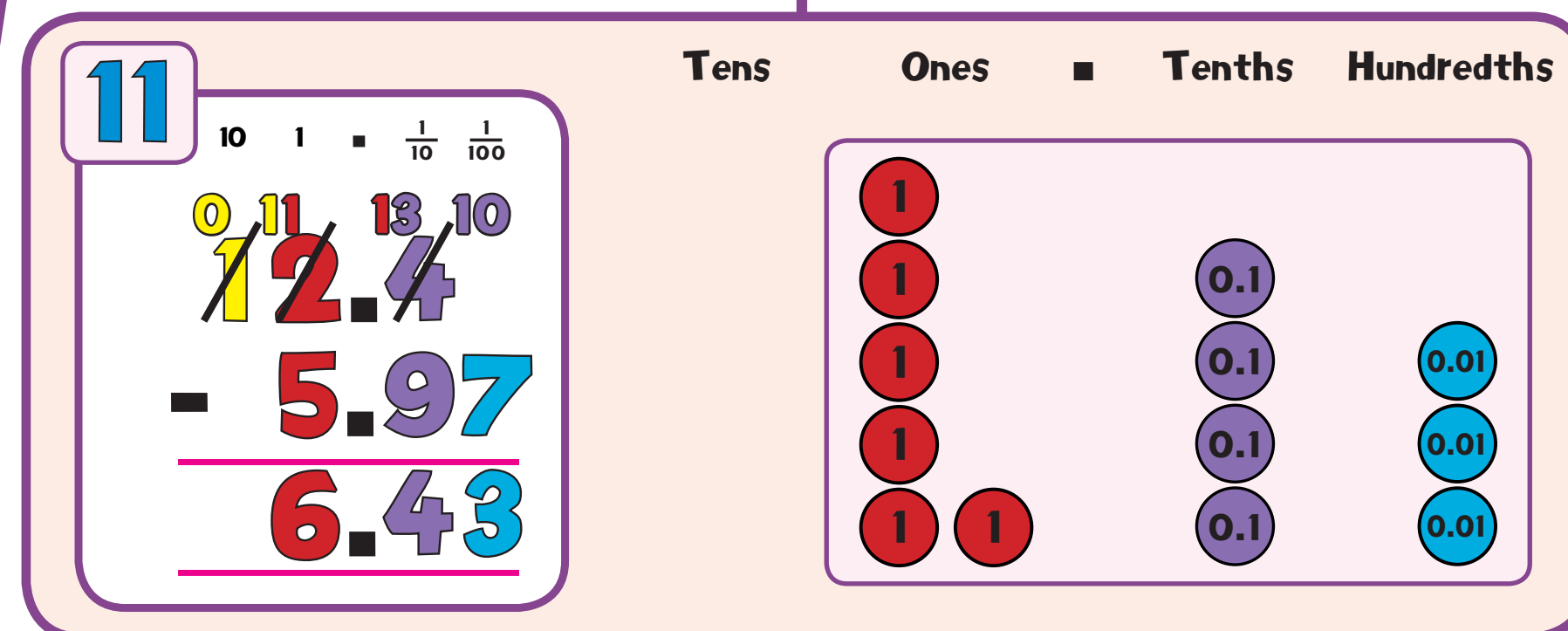
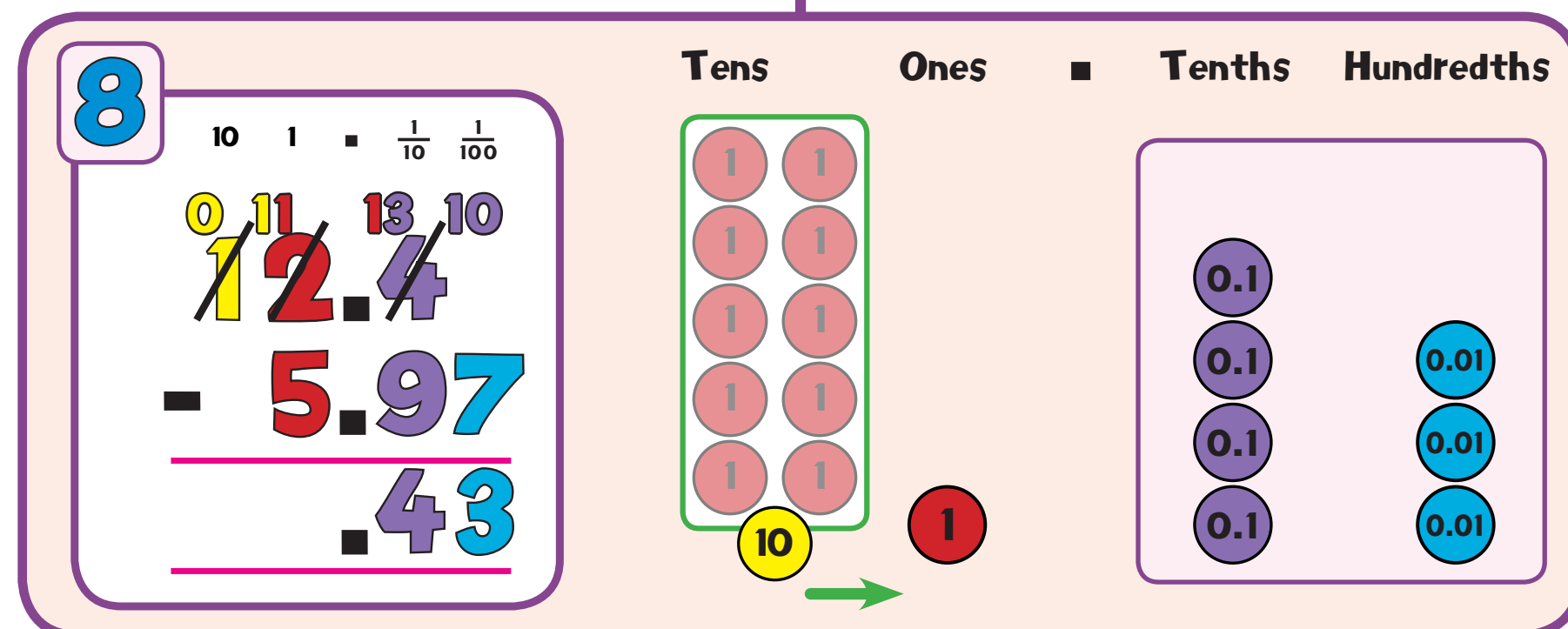
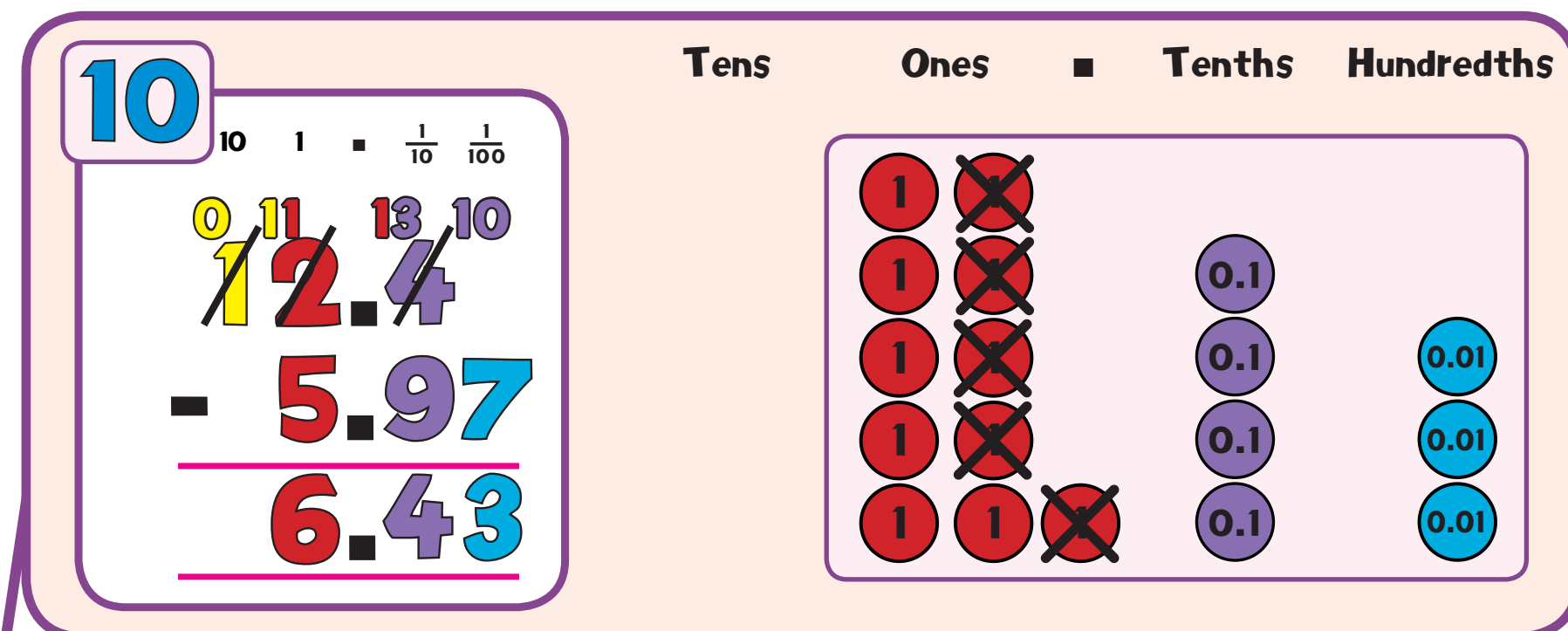
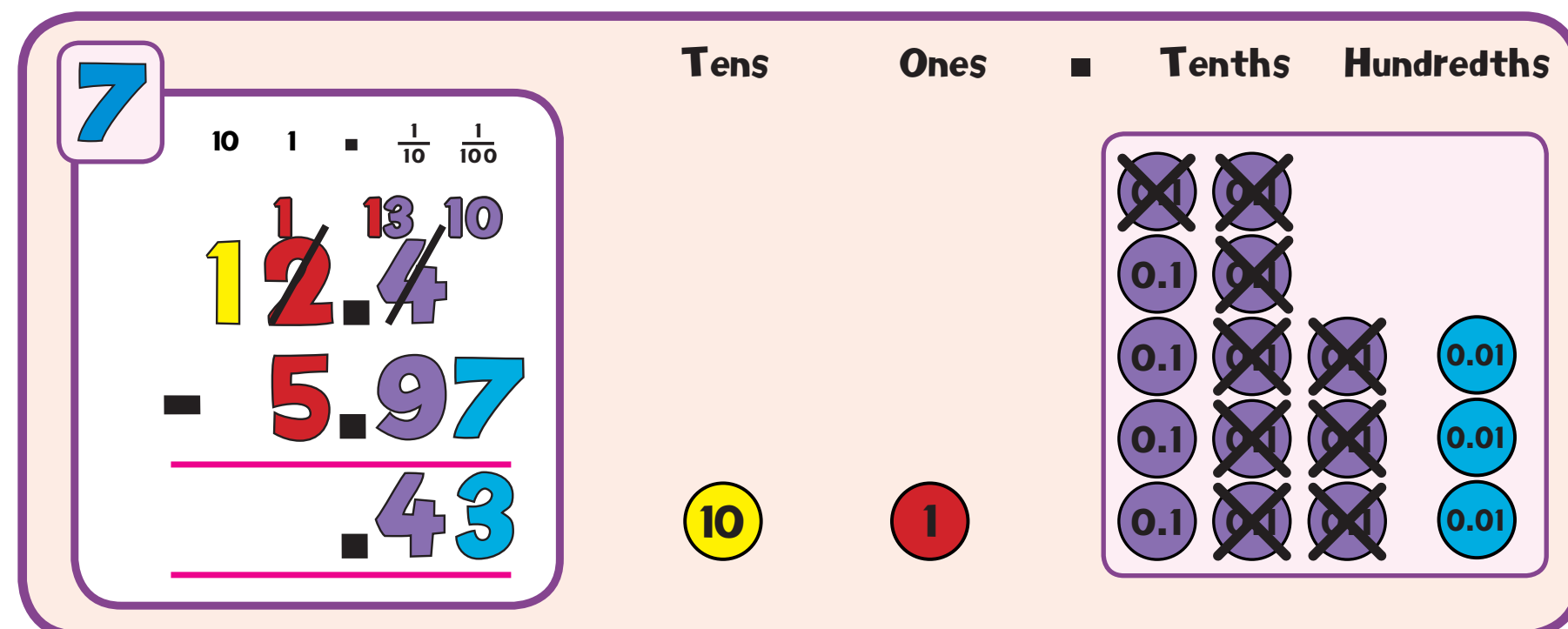
$$72.43 - 47.85 = 24.58$$



$$12.4 - 5.97 = 6.43$$



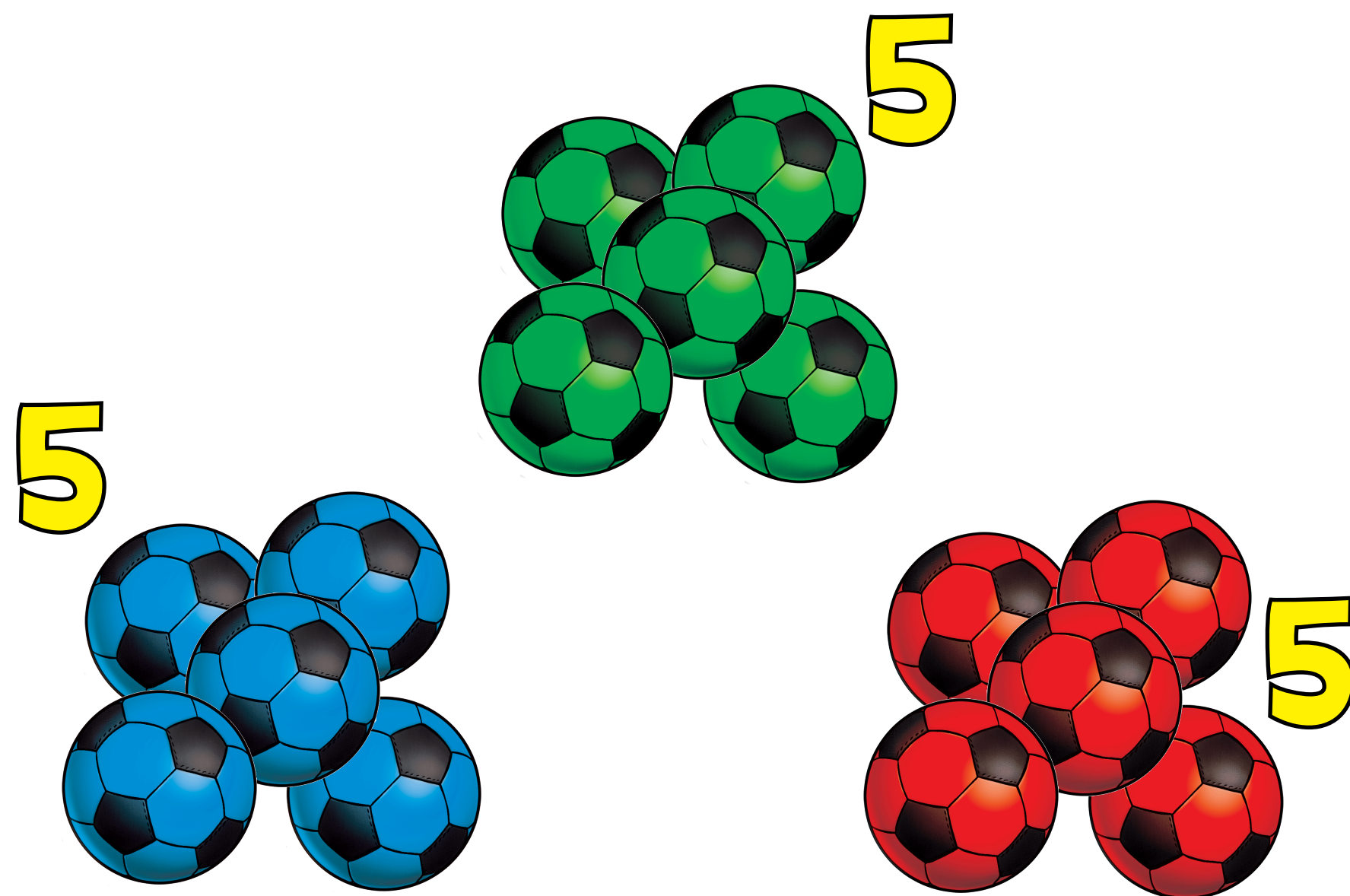
$$12.4 - 5.97 = 6.43$$



CPA Models of Multiplication

reasoning

Repeated Addition



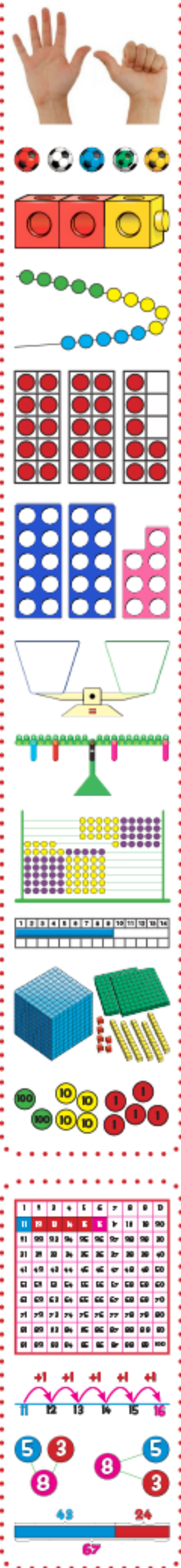
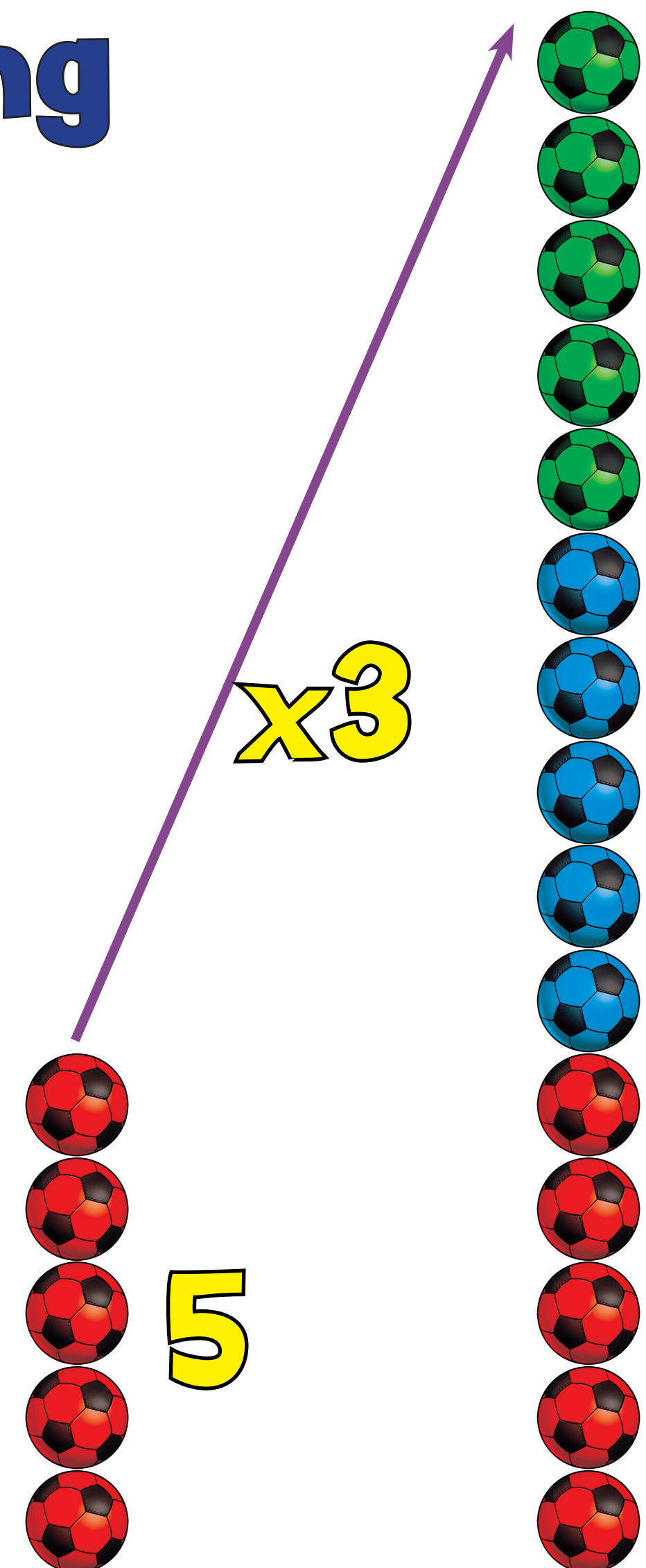
$$5 + 5 + 5 = 15$$

“I can fit 5 footballs in a bag and I’ve brought 3 full bags. How many footballs do I have with me?” “15”

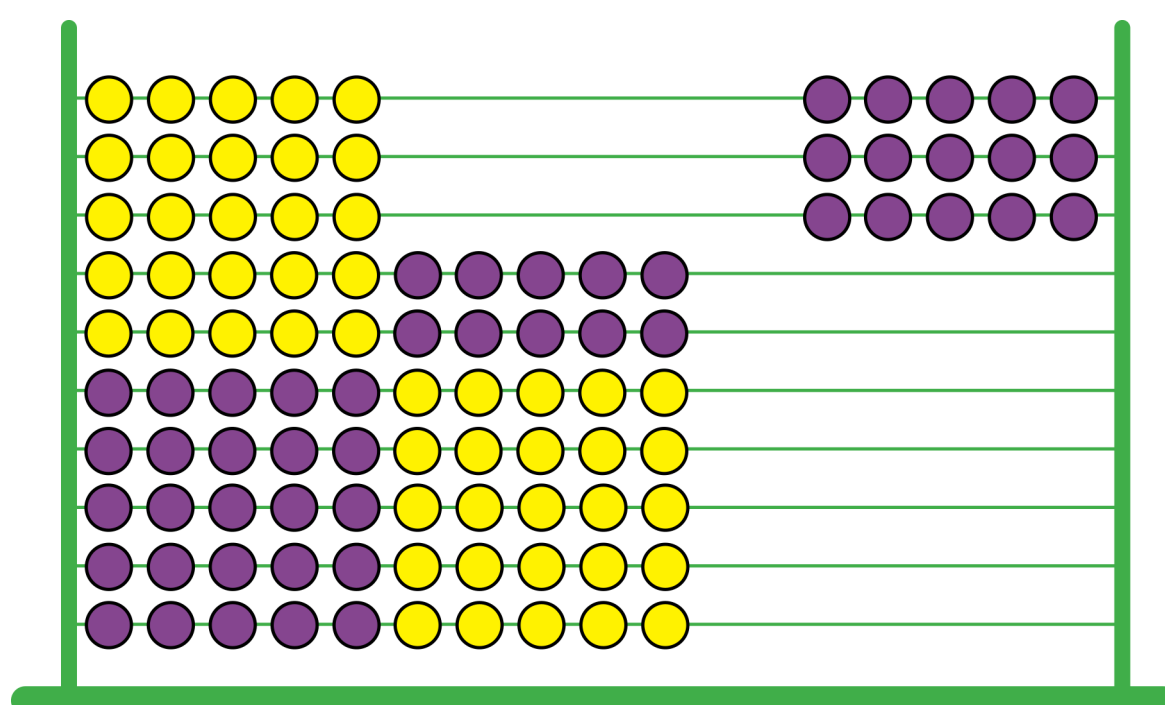
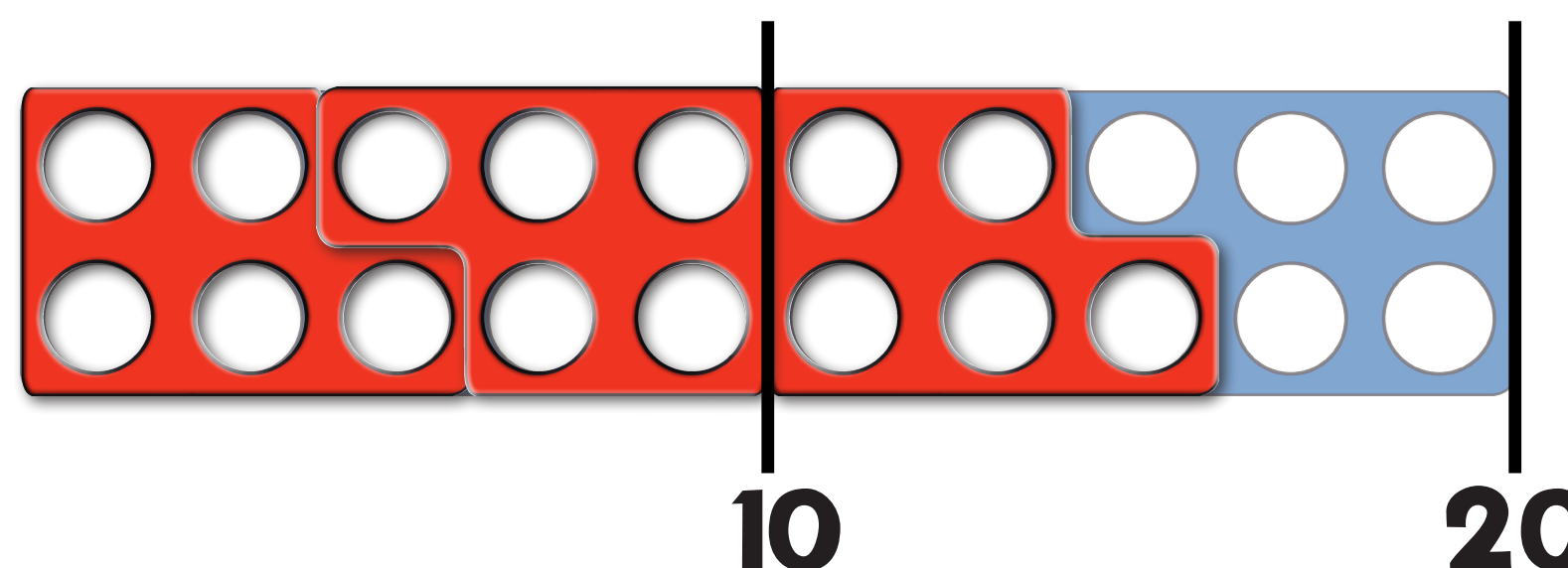
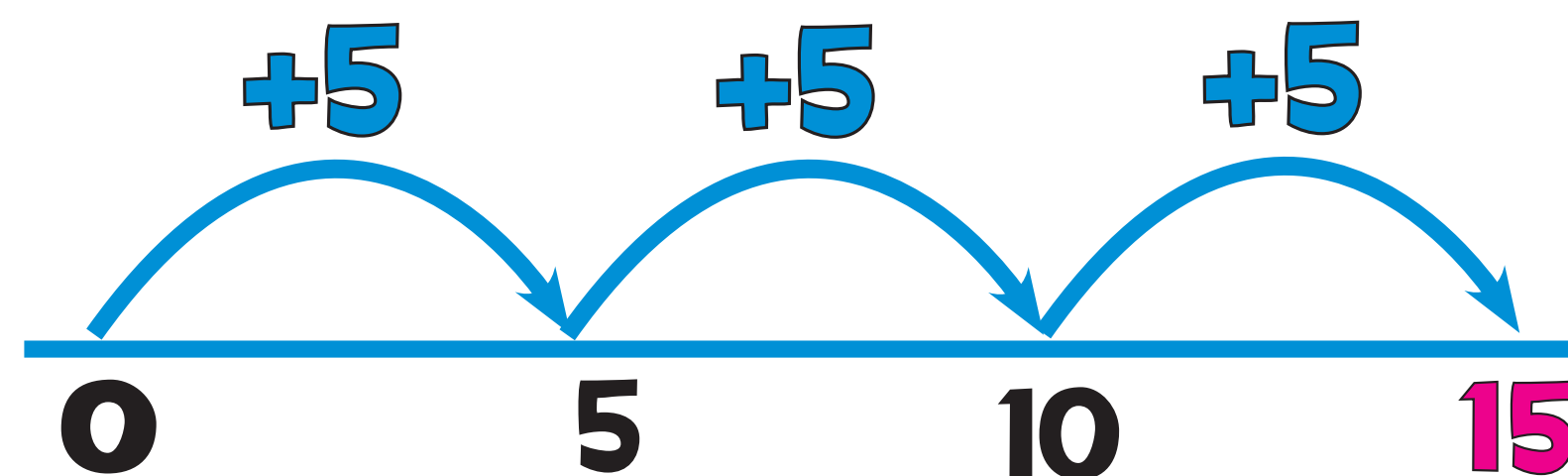
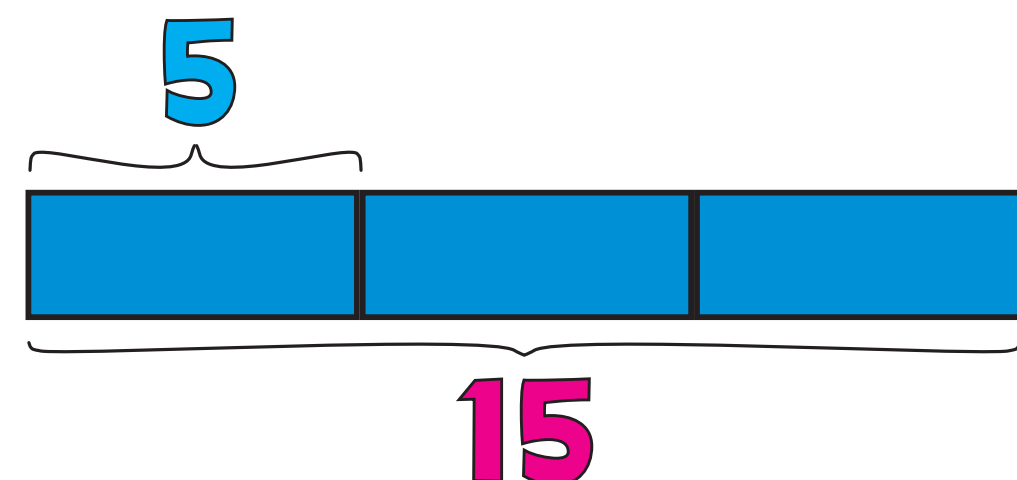
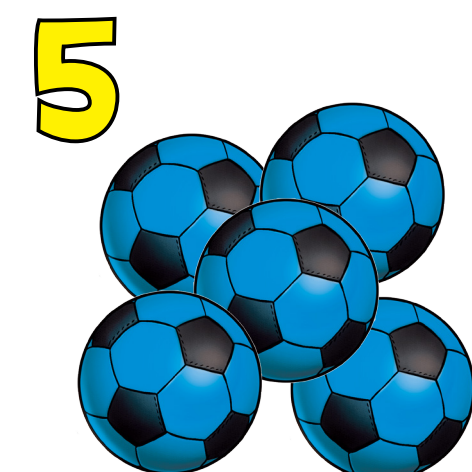
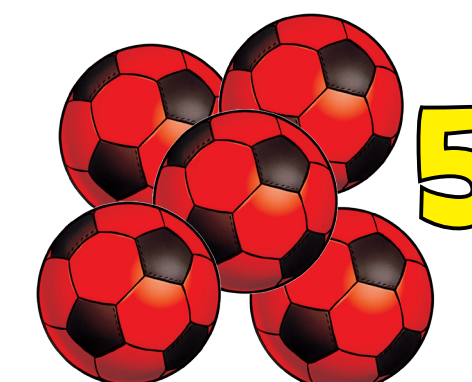
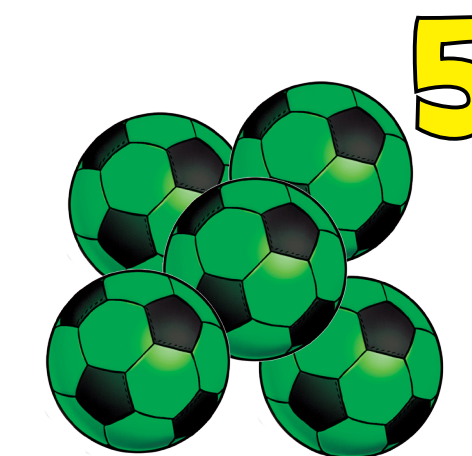
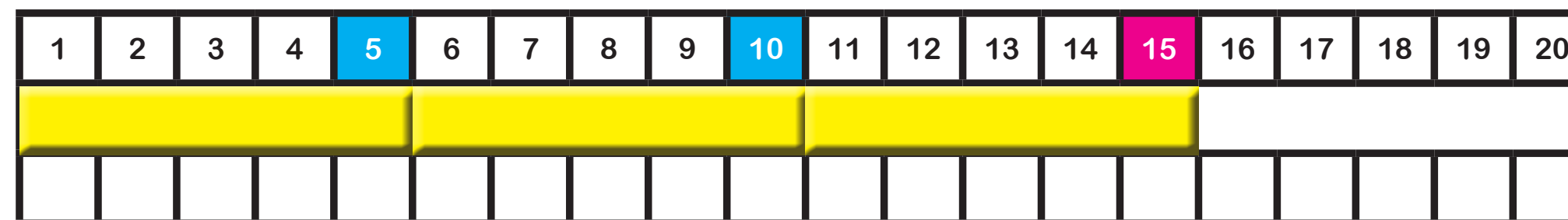
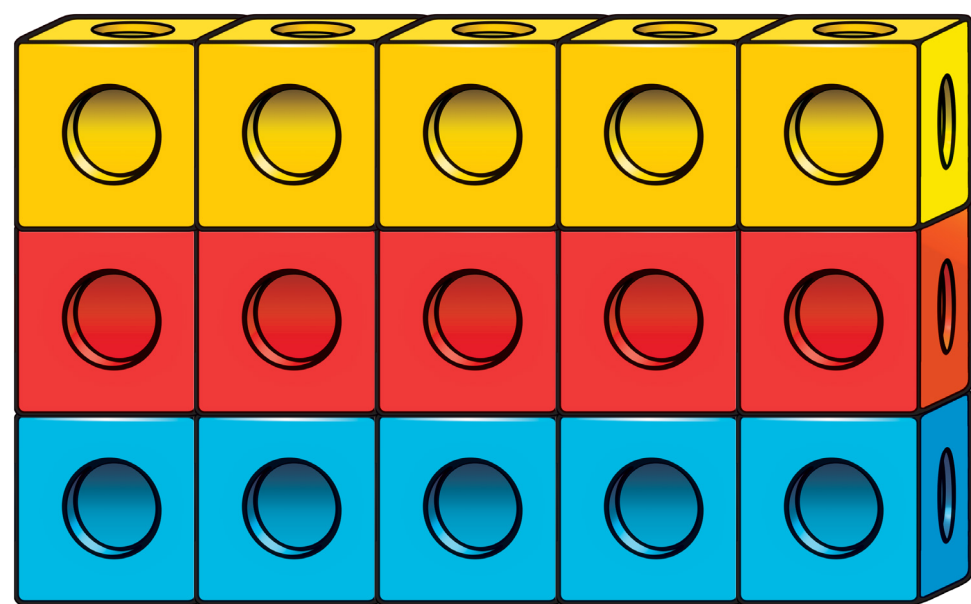
Scaling

“My tower of football stickers was 5 high, but now it’s 3 times as big! How many stickers in my tower?” “15”

$$5 \times 3 = 15$$

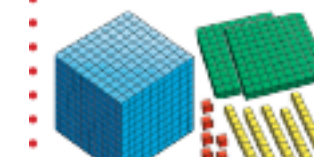
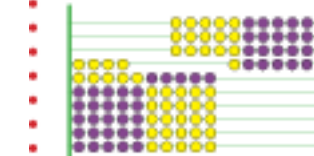
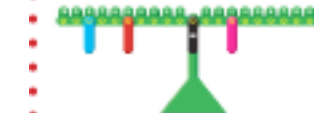
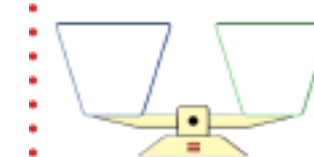
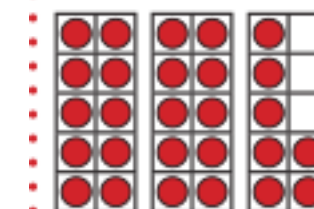
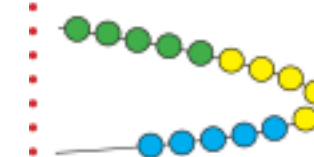


$$5 \times 3 = 15$$



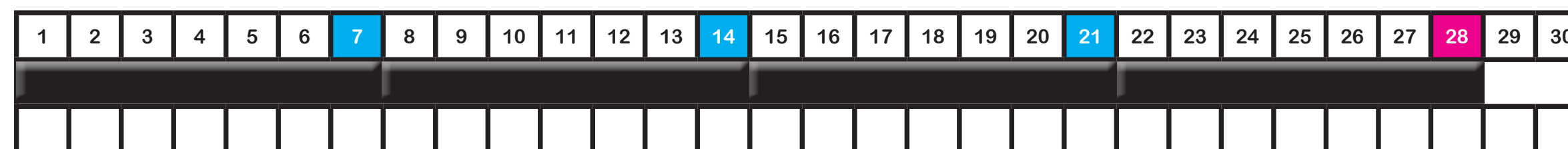
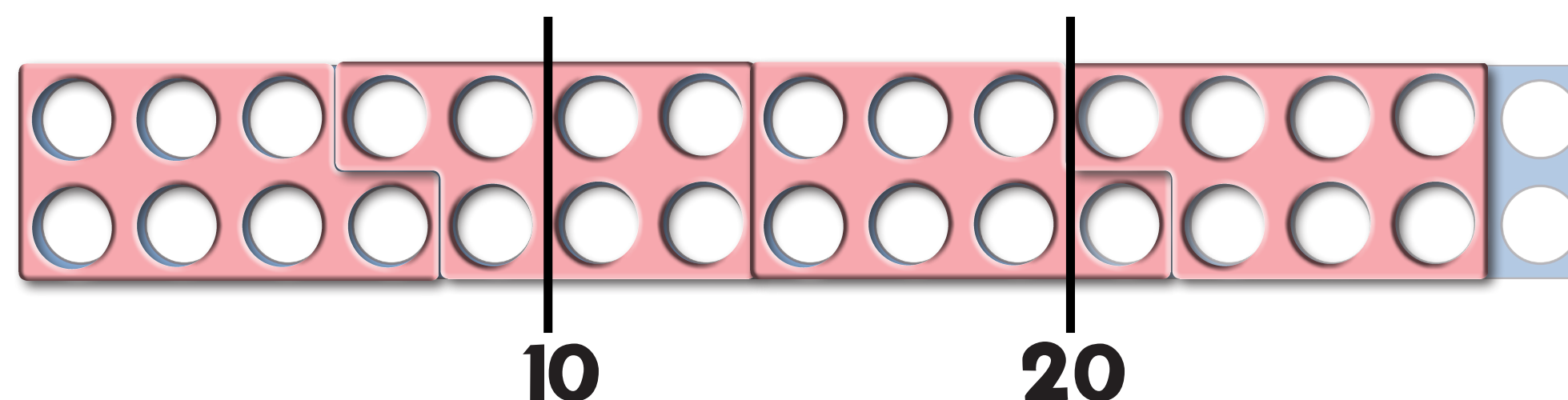
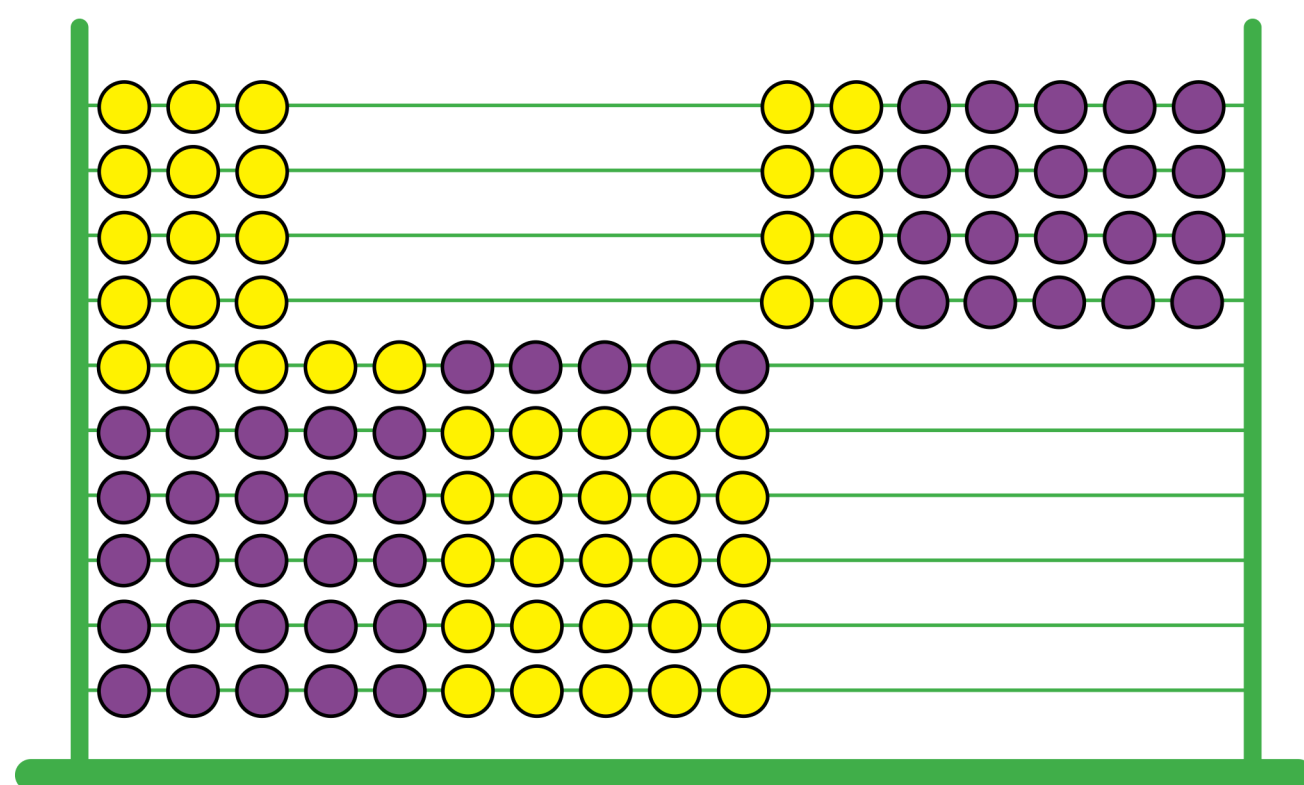
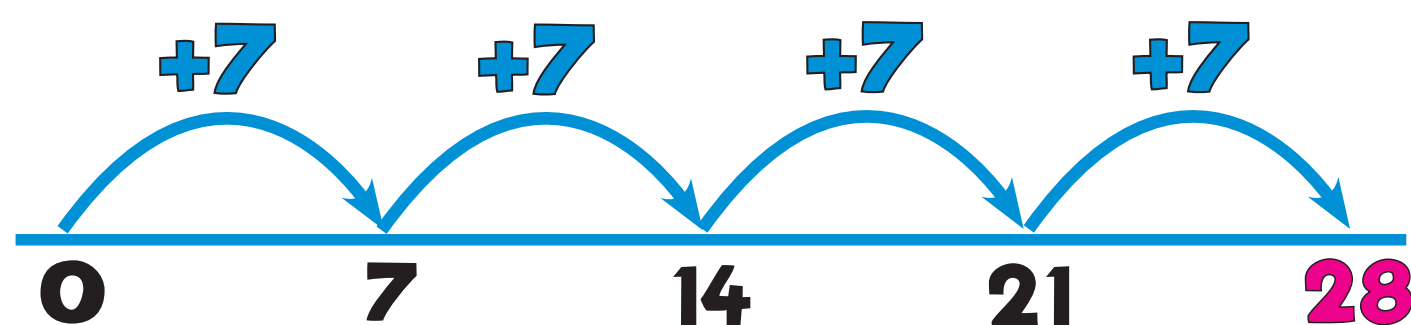
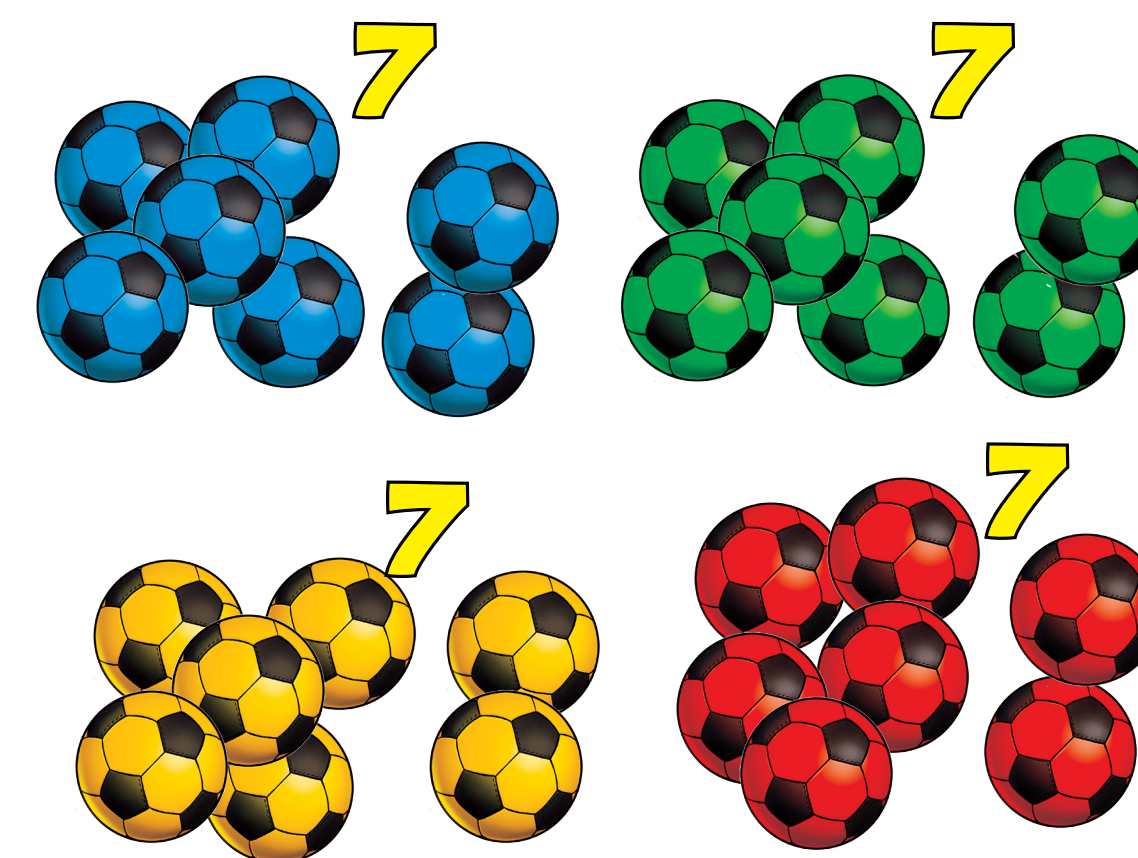
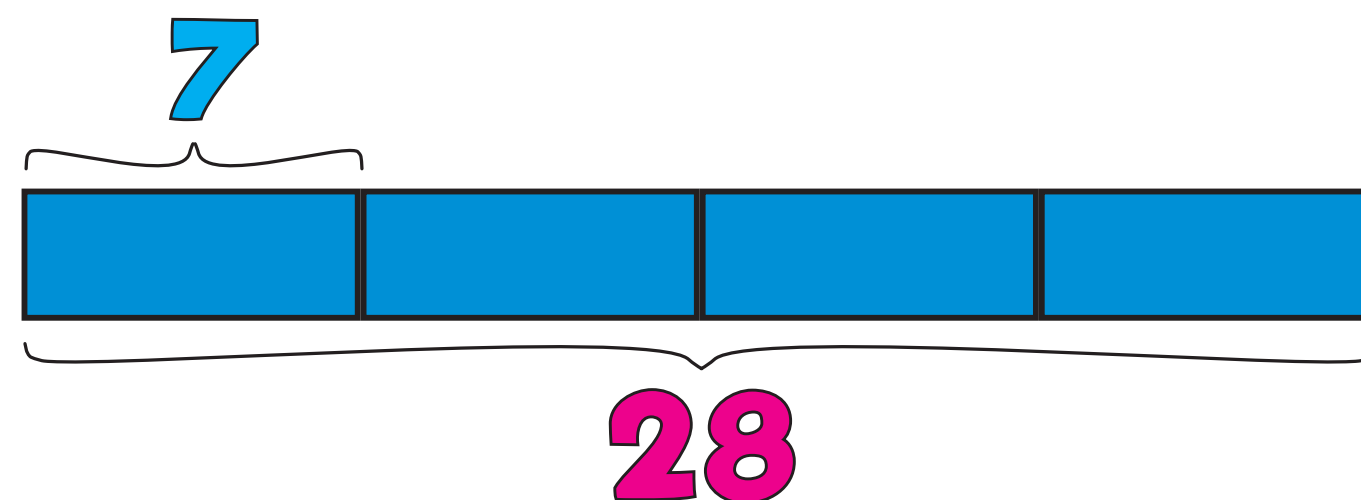
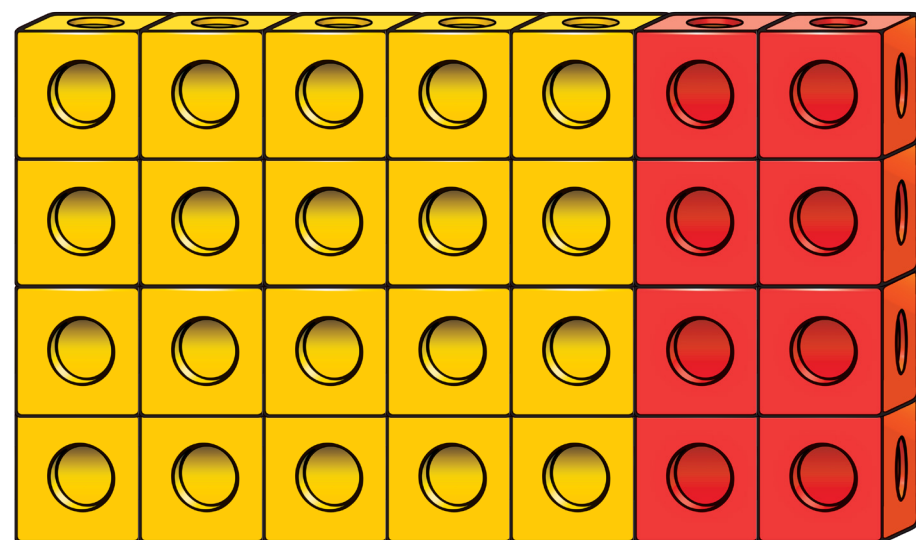
$$5 \times 3 = 5 + 5 + 5 = 15$$

“5 times 3”
“5 multiplied by 3”



reasoning

7 x 4 = 28

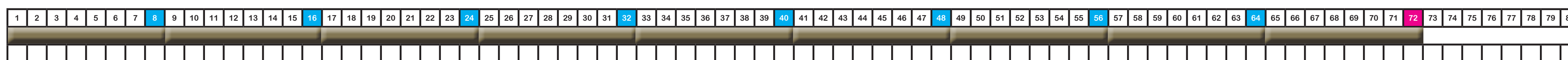
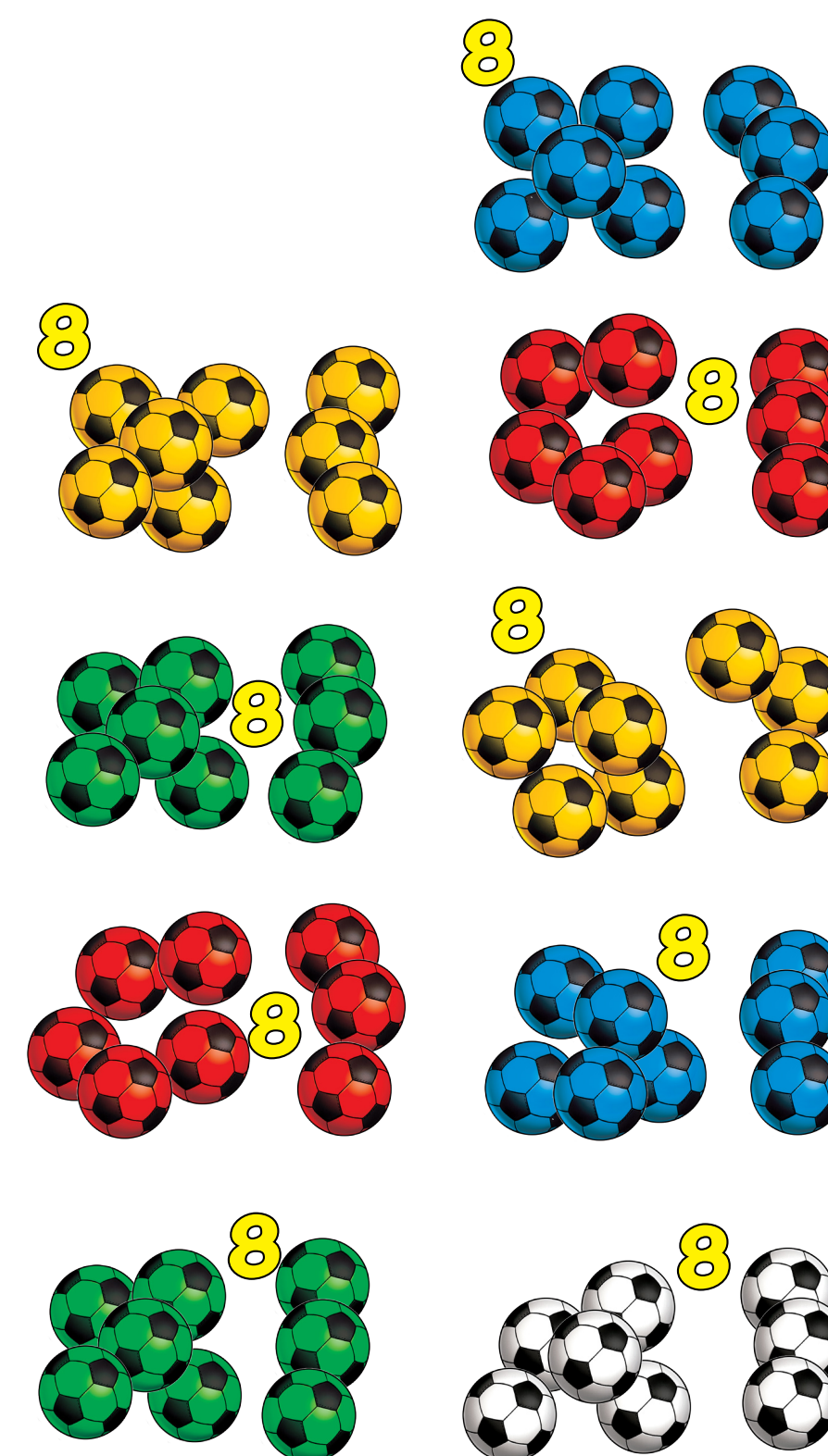
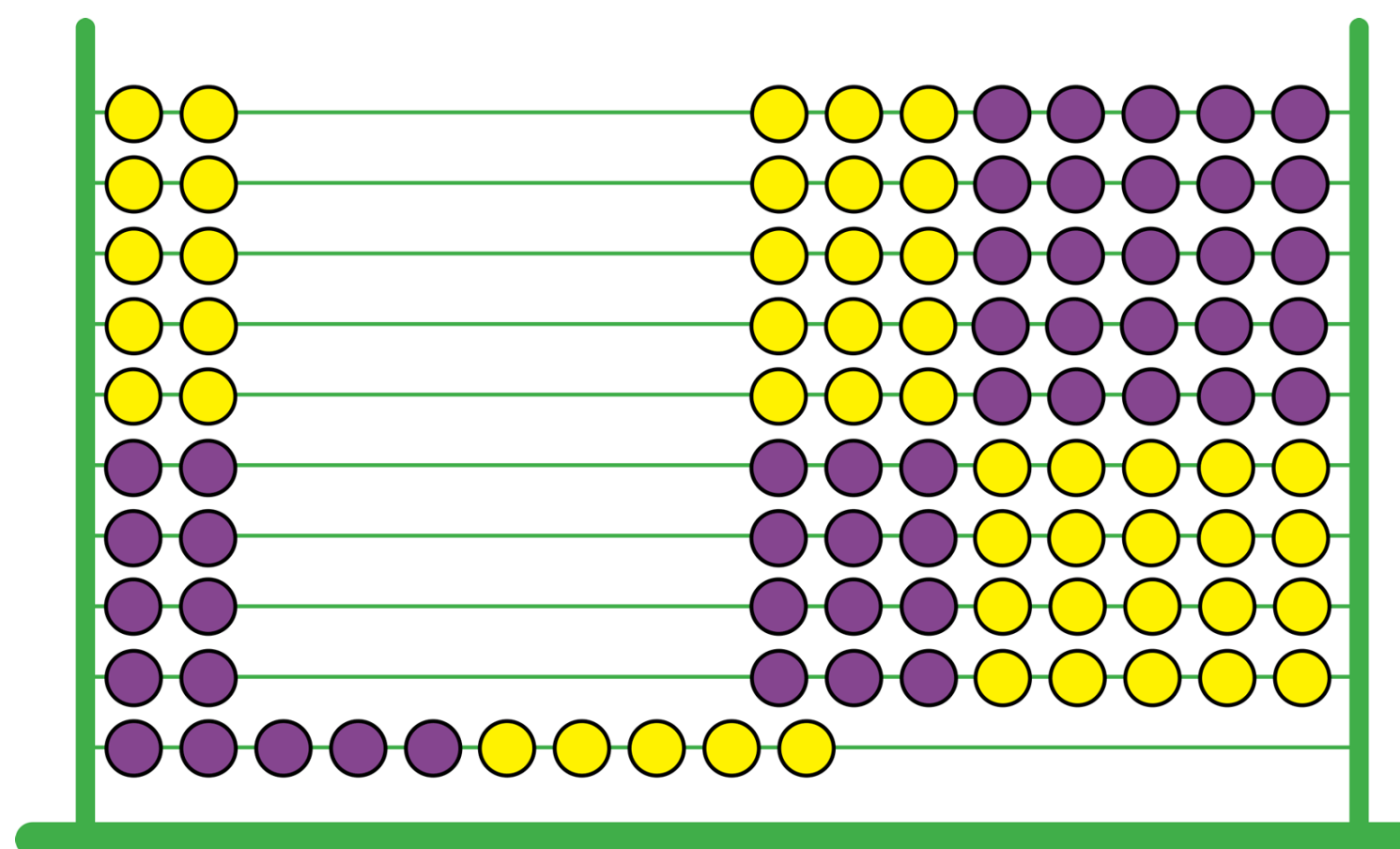
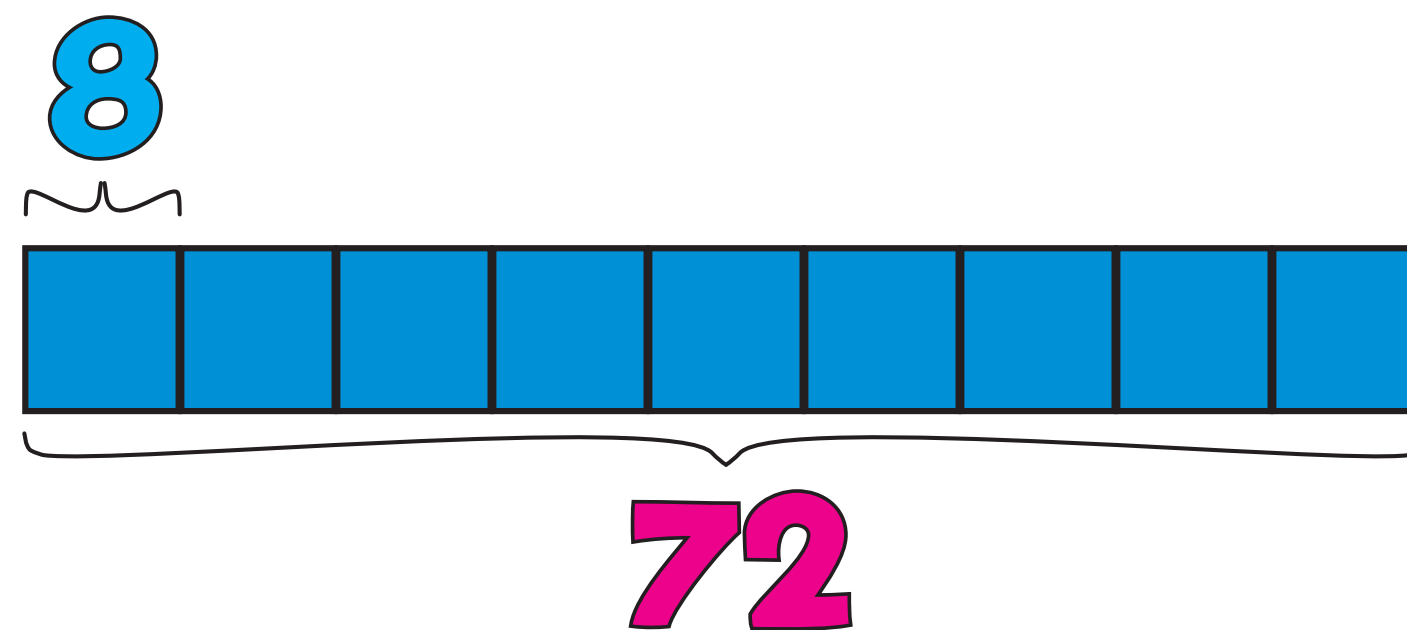
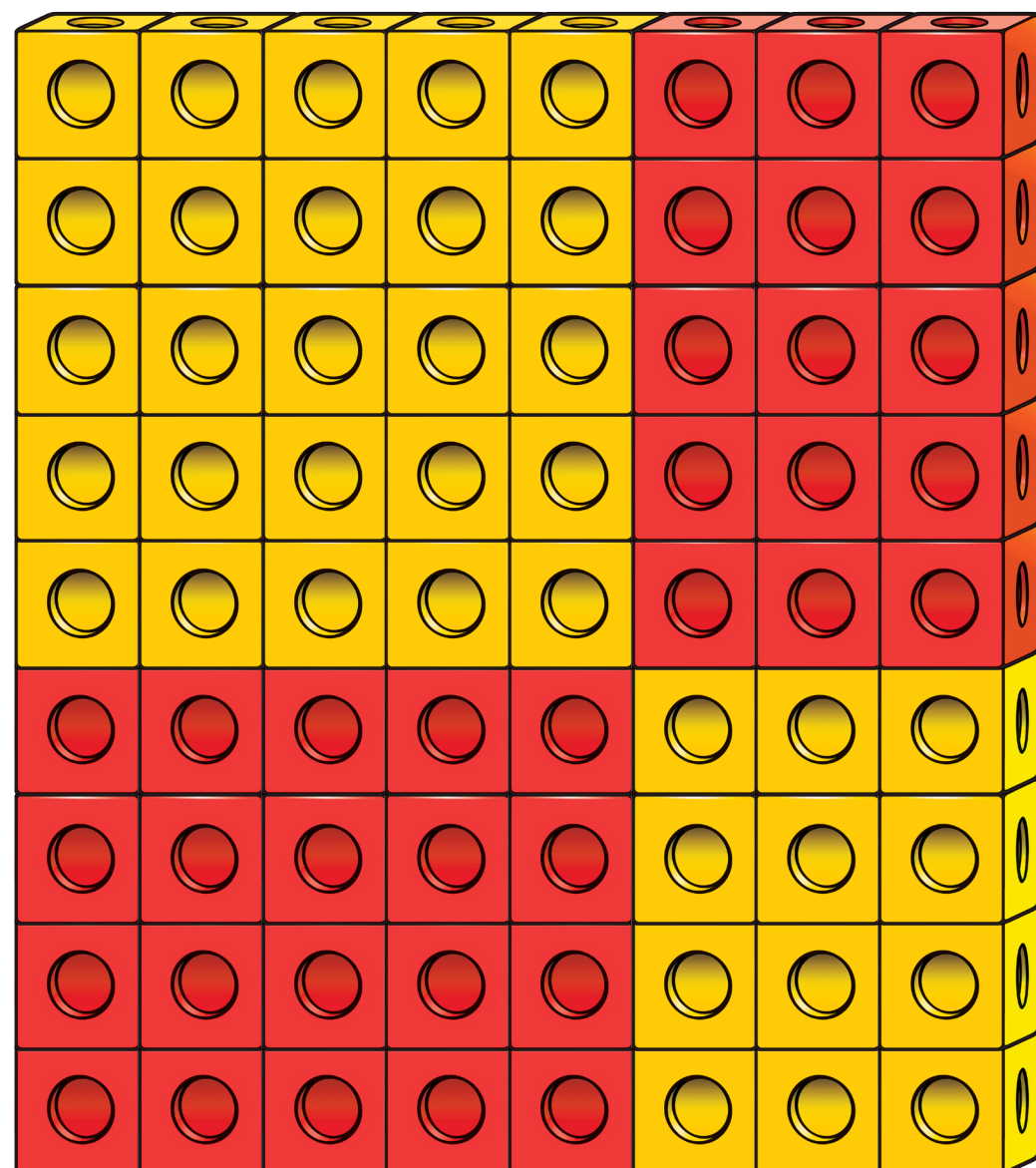


7 x 4 = 7 + 7 + 7 + 7 = 28

“7 times 4”
“7 multiplied by 4”



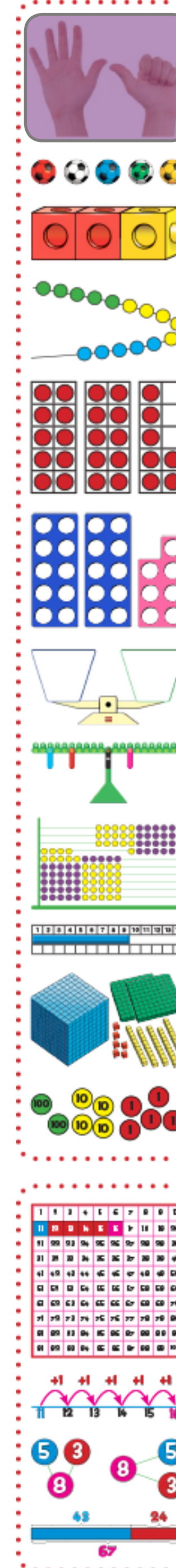
$$8 \times 9 = 72$$



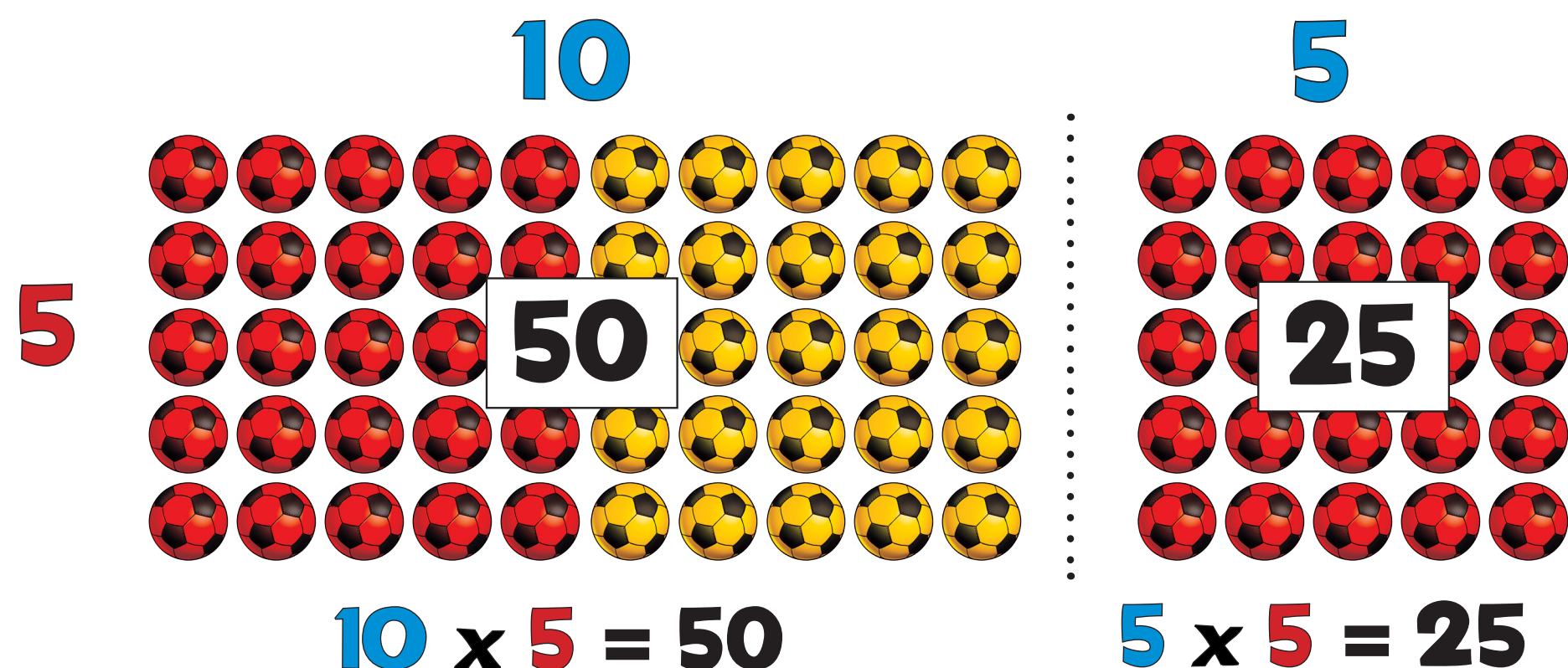
“8 multiplied by 9”

“8 times 9”

$$8 \times 9 = 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 = 72$$

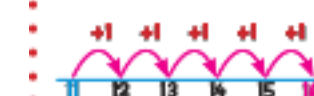
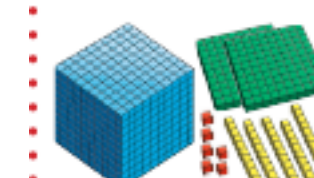
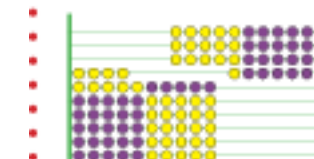
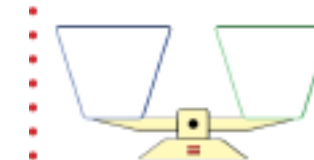
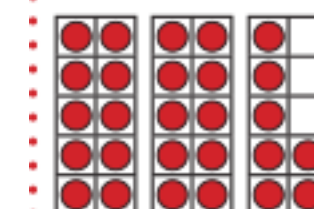
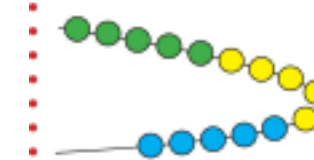
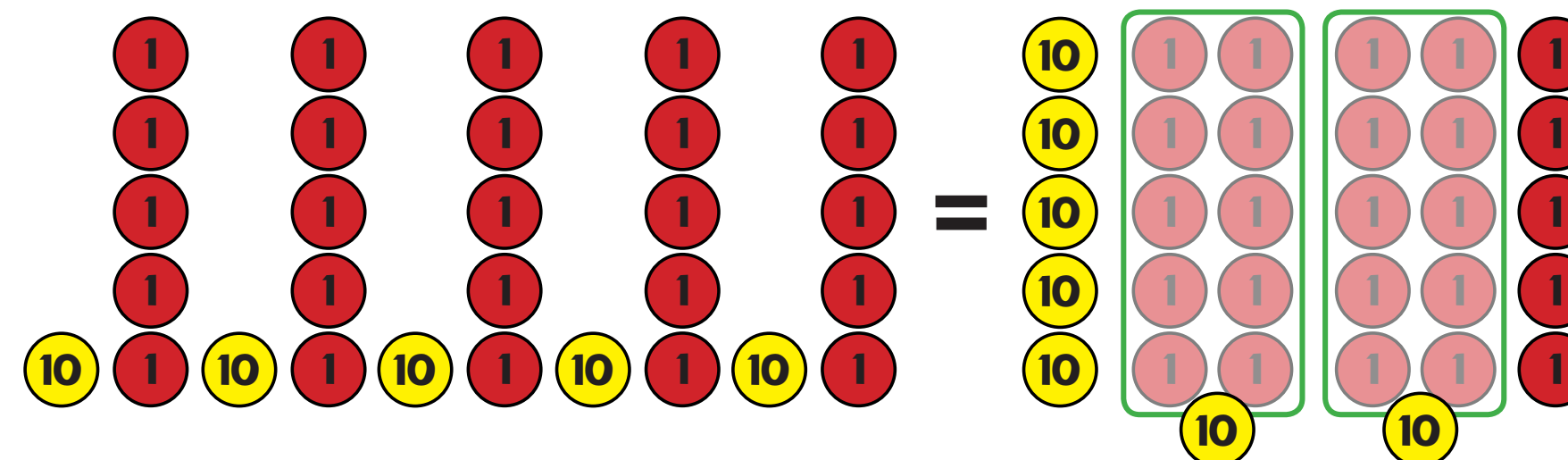
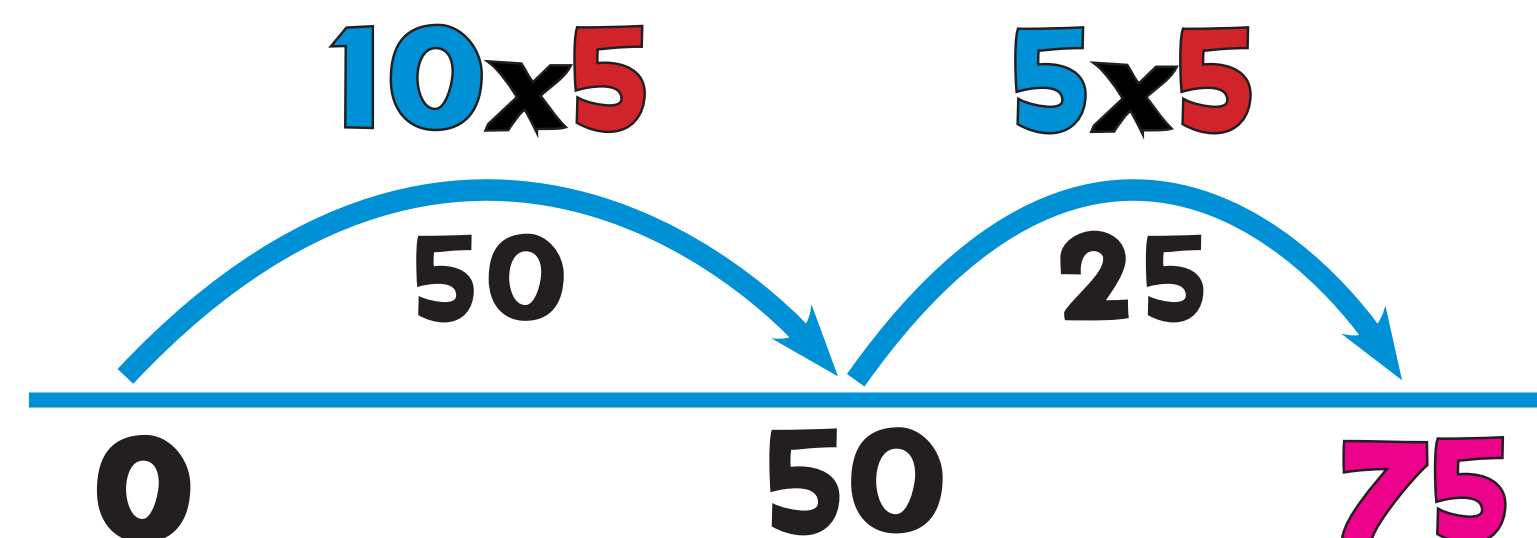
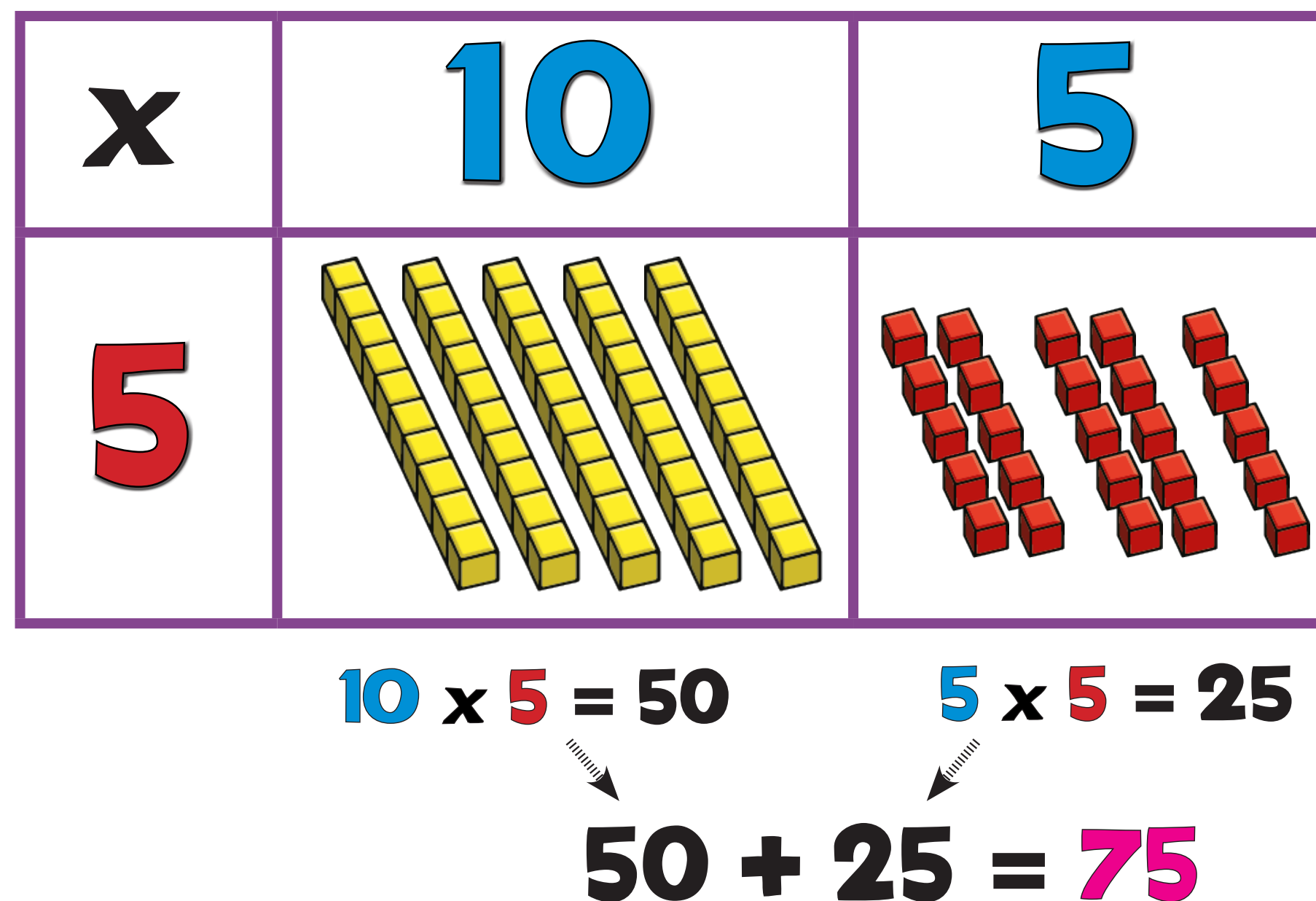


$$15 \times 5 = 75$$

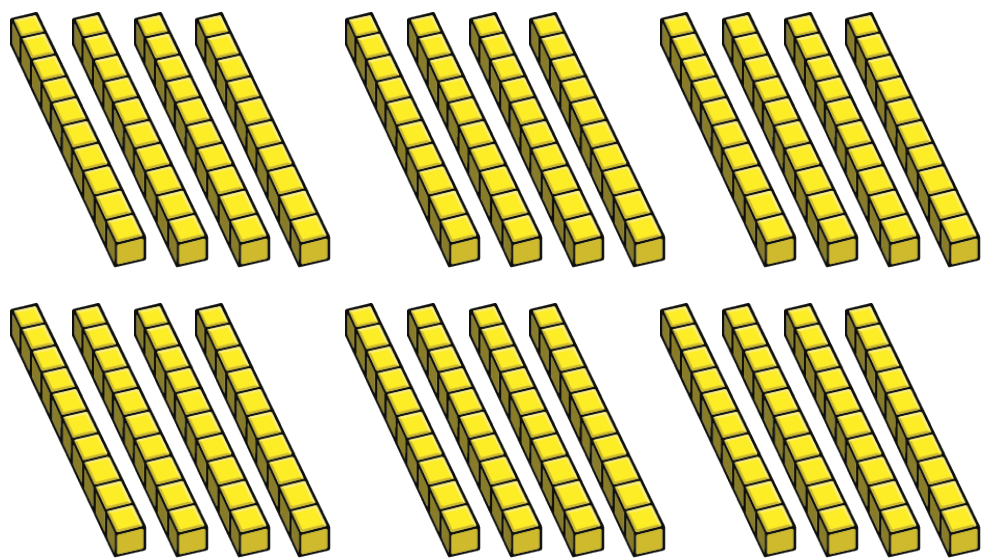
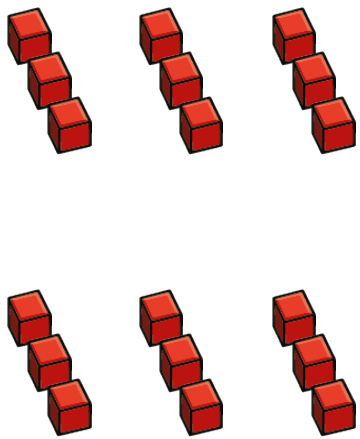


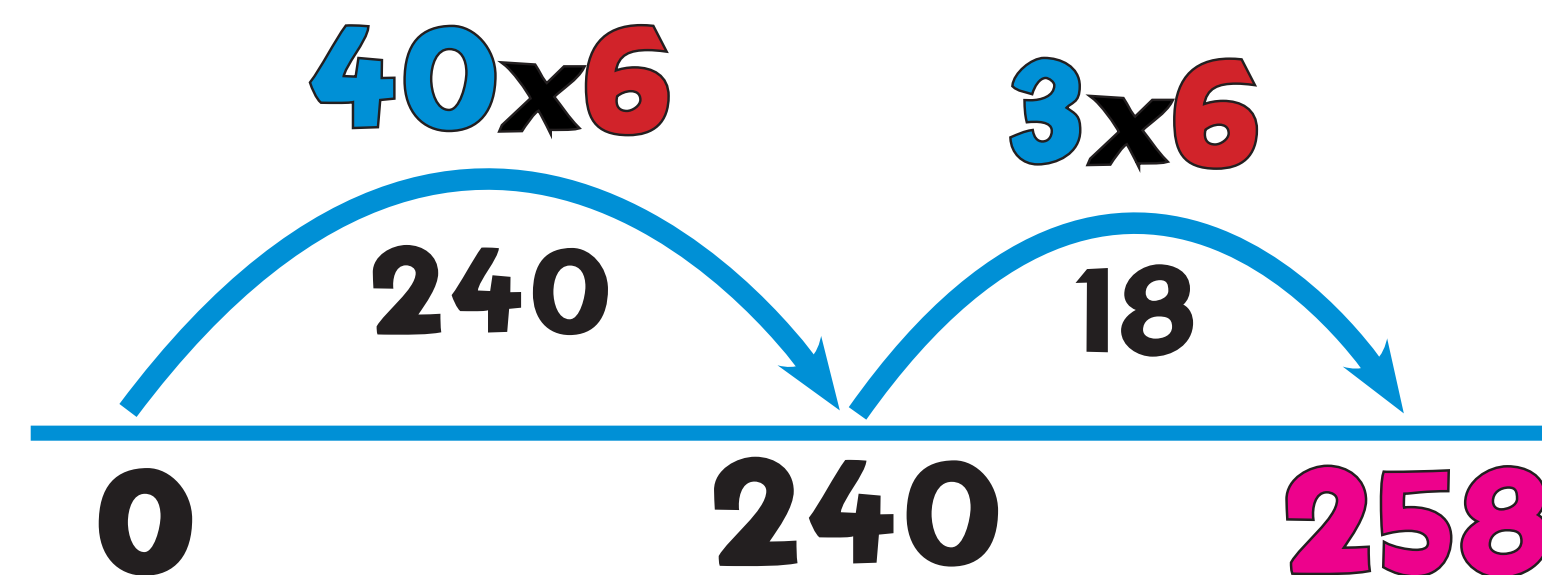
x	10	5
5	50	25

$50 + 25 = 75$

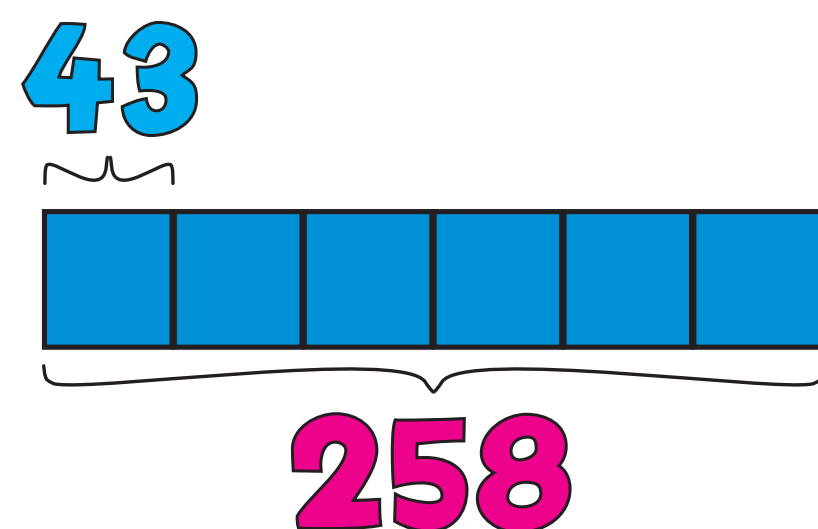


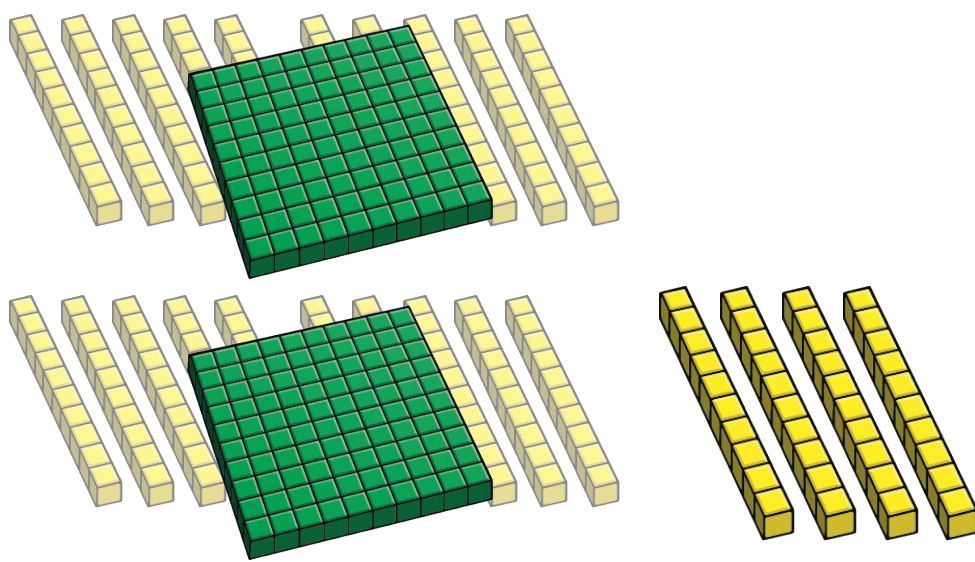
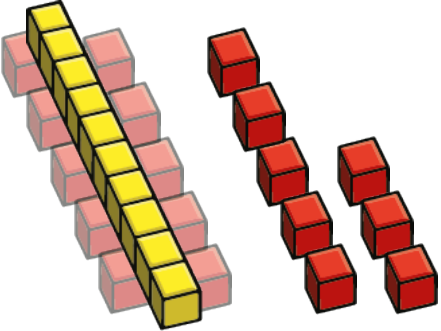
$$43 \times 6 = 258$$

x	40	3
6		

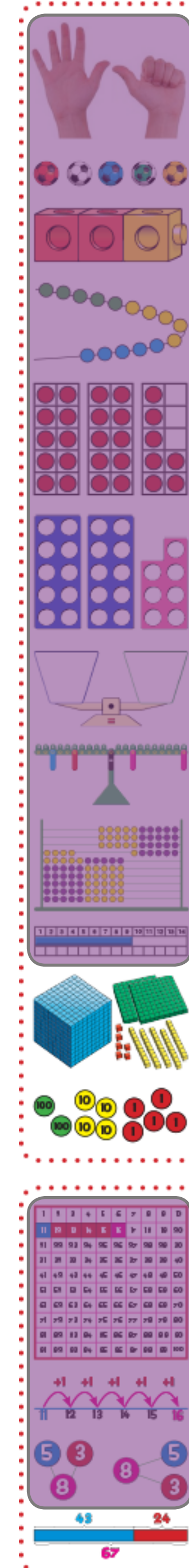


x	40	3
6	240	18

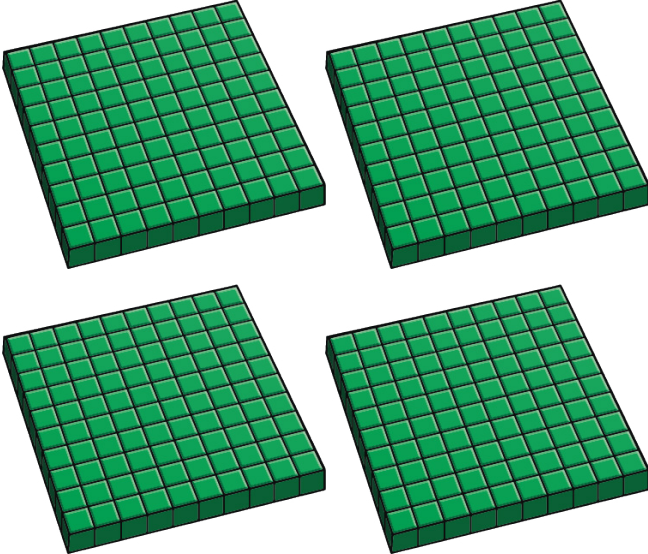
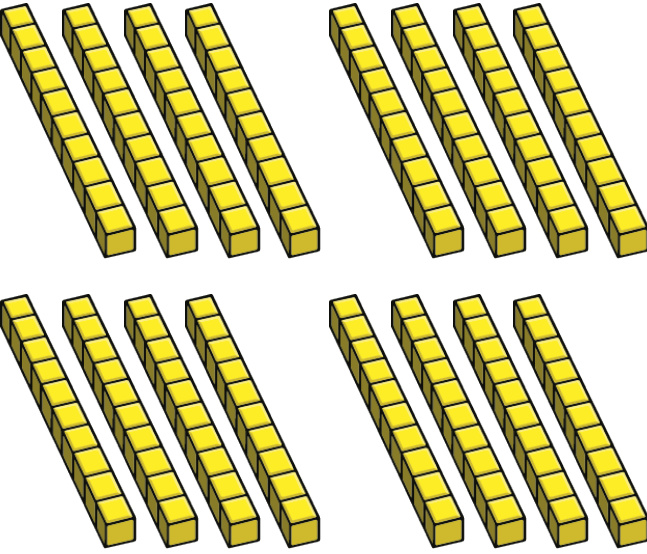
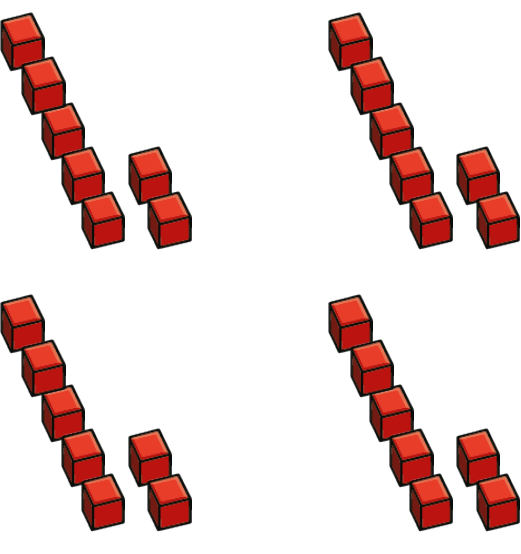


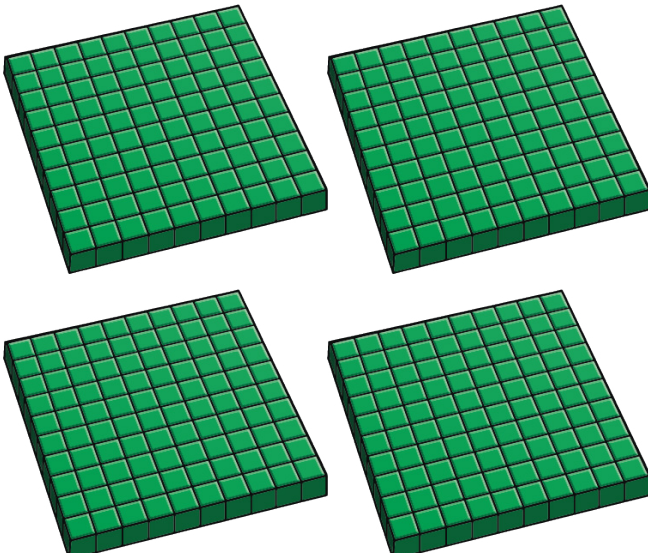
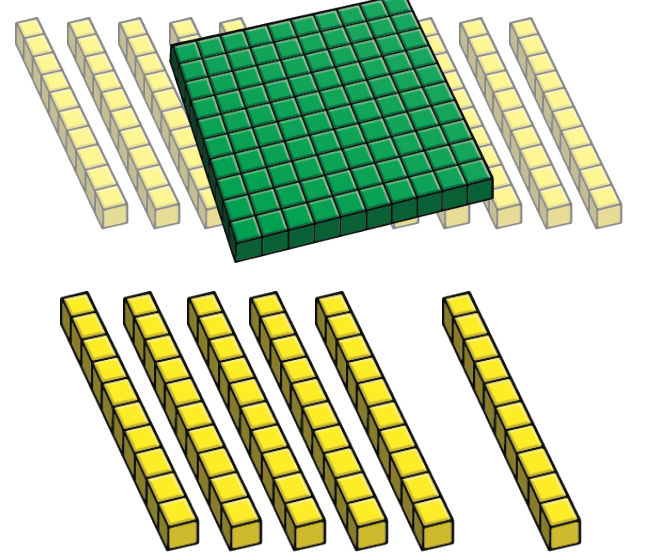
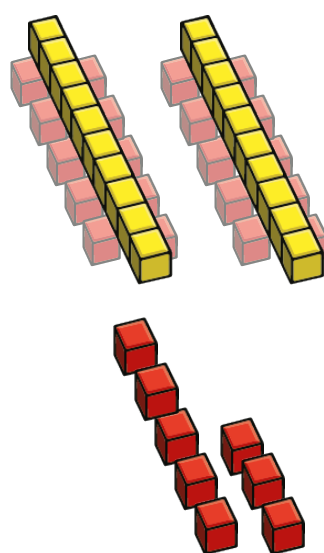
x	40	3
6		

$$240 + 18 = 258$$

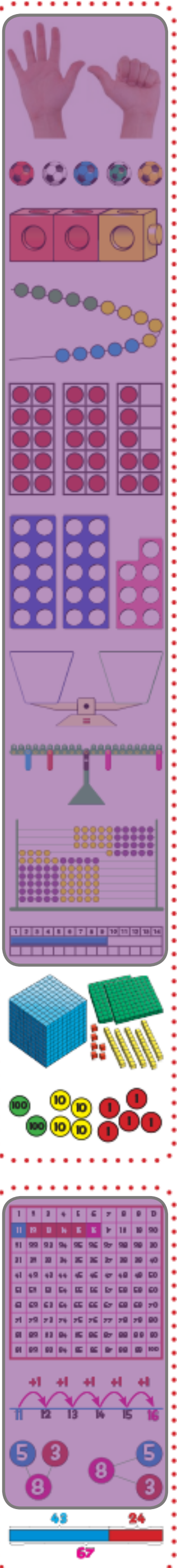


$$147 \times 4 = 588$$

x	100	40	7
4			

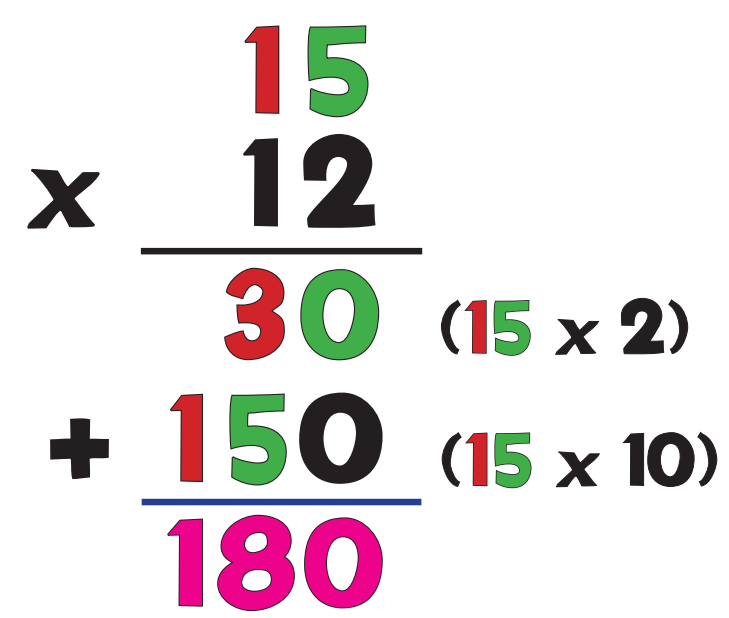
x	100	40	7
4			

$$400 + 160 + 28 = 588$$



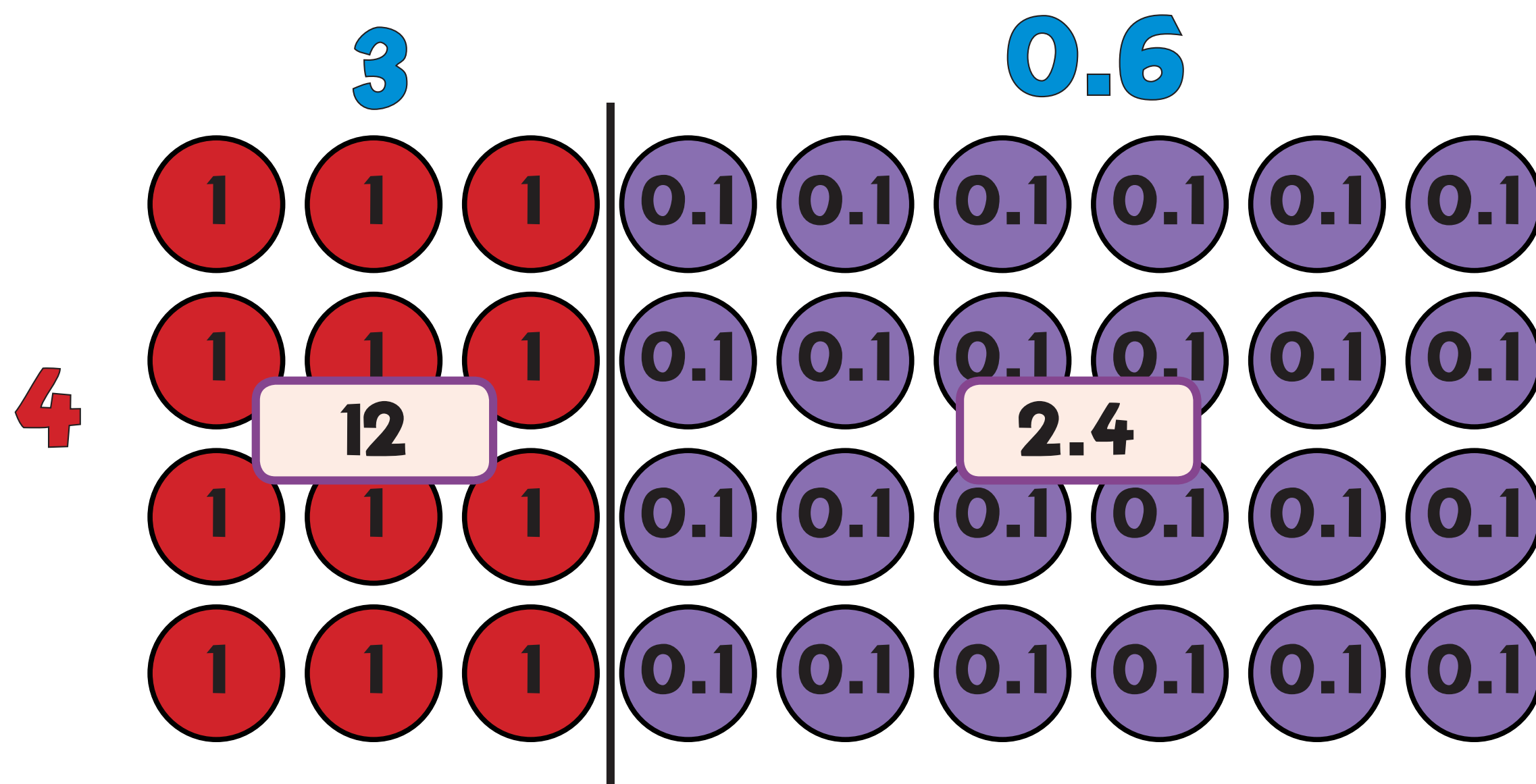


CPA
reasoning

$$15 \times 12 = 180$$


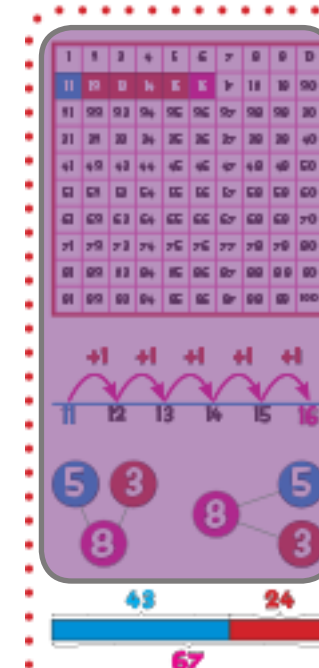
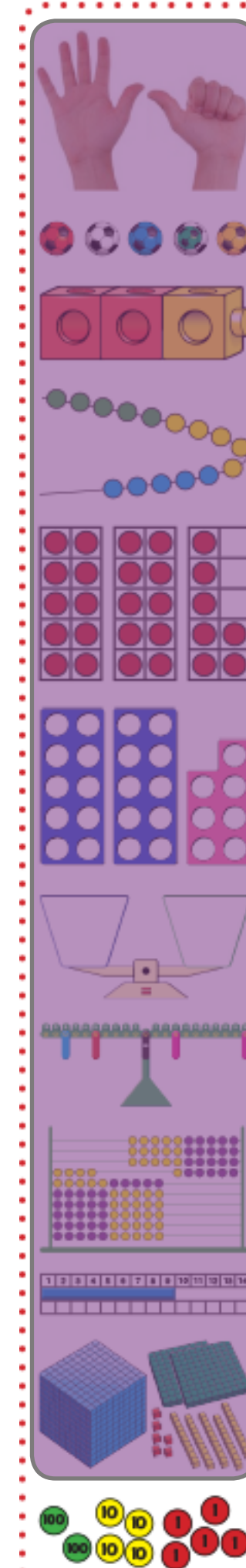
x	10	5
10	100	50
2	20	10

$$3.6 \times 4 = 14.4$$

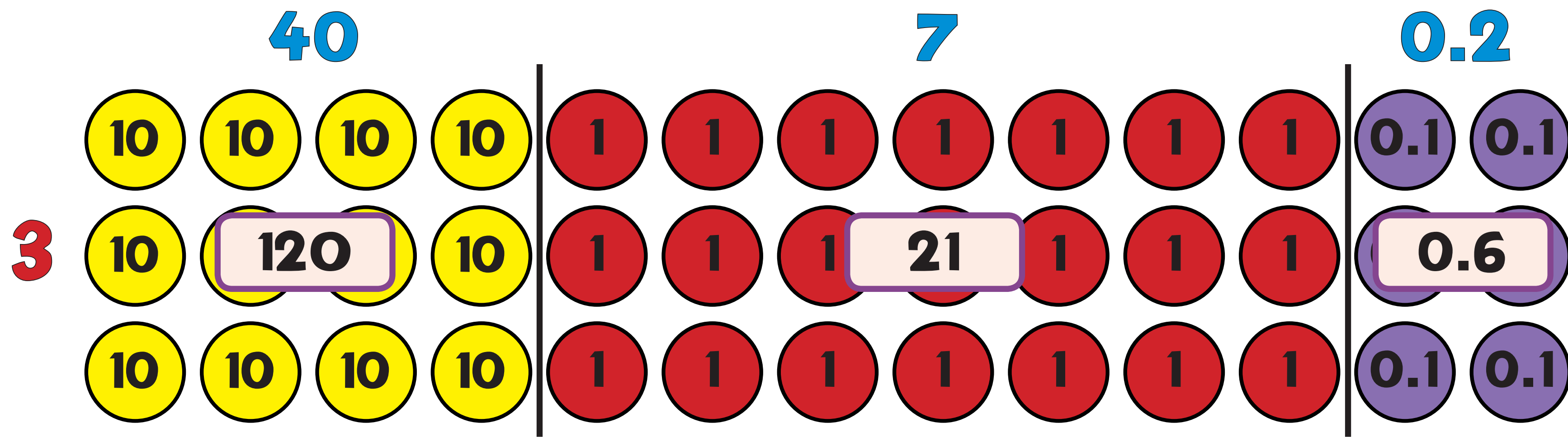


$$\begin{array}{r} 10 \quad 1 \quad \frac{1}{10} \\ 3.6 \\ \times 4 \\ \hline 14.4 \\ \hline 2 \end{array}$$

x	3	0.6
4	12	2.4

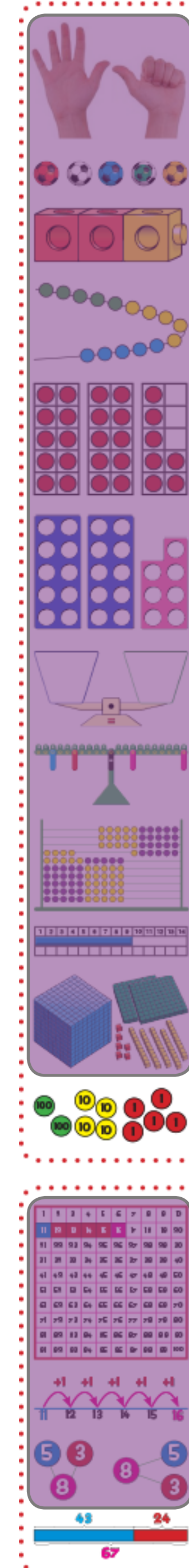


$$47.2 \times 3 = 141.6$$

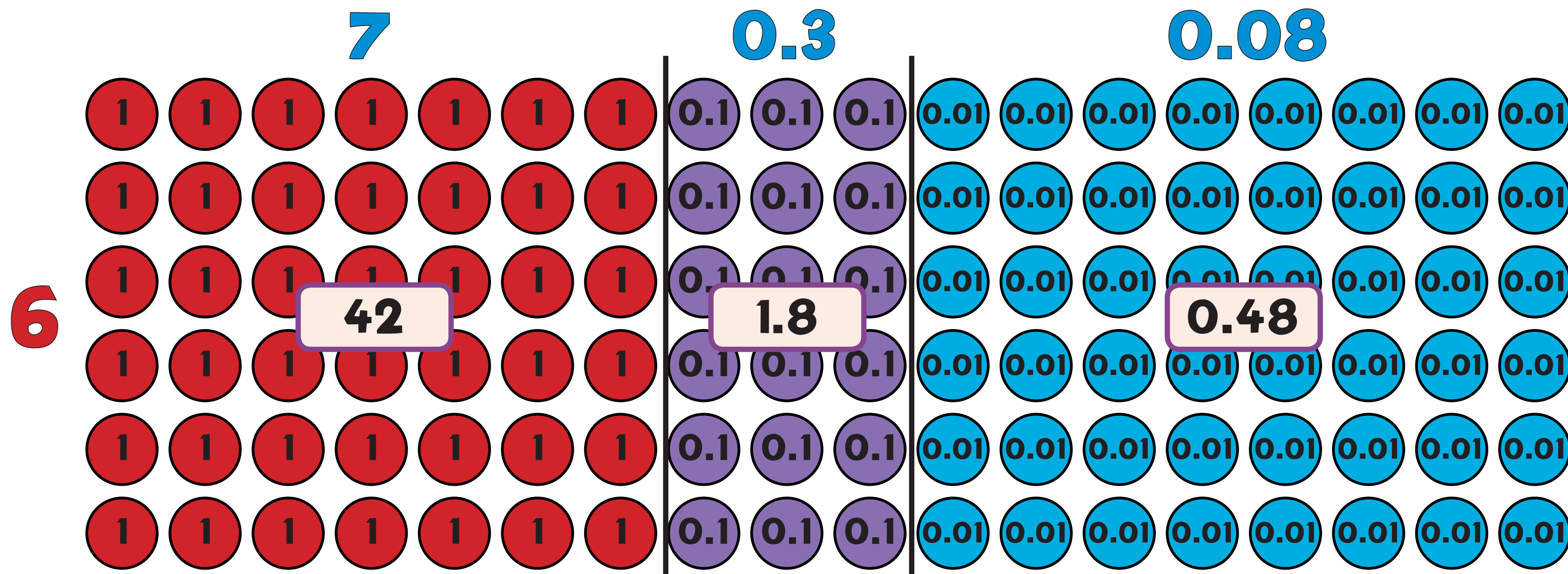


x	40	7	0.2
3	120	21	0.6

$$\begin{array}{r} 100 \quad 10 \quad 1 \quad \frac{1}{10} \\ 47.2 \\ \times 3 \\ \hline 141.6 \\ \hline 2 \end{array}$$

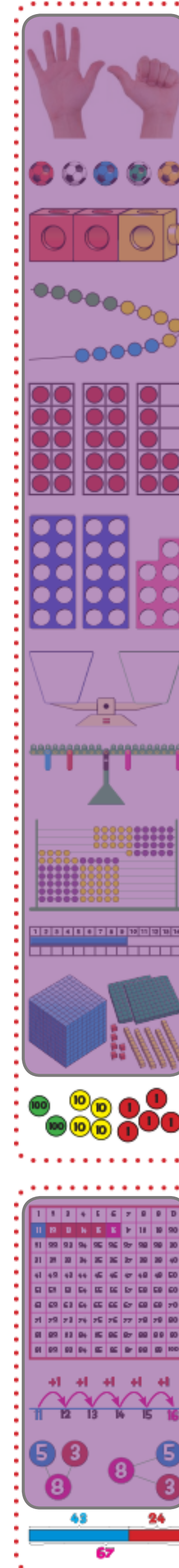


$$7.38 \times 6 = 44.28$$



x	7	0.3	0.08
6	42	1.8	0.48

$$\begin{array}{r} 10 \quad 1 \quad \frac{1}{10} \quad \frac{1}{100} \\ 7.38 \\ \times 6 \\ \hline 44.28 \\ \hline 4 \quad 2 \quad 4 \end{array}$$





Sharing



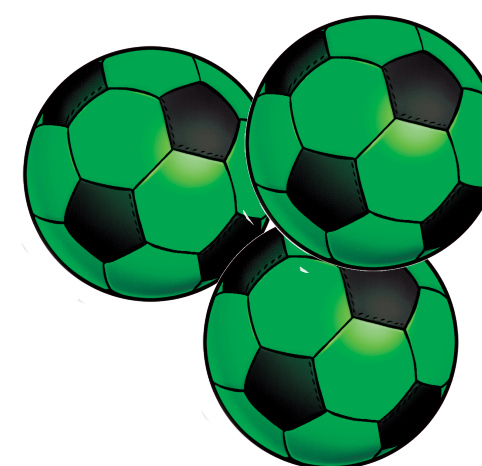
**5 in each
group!**



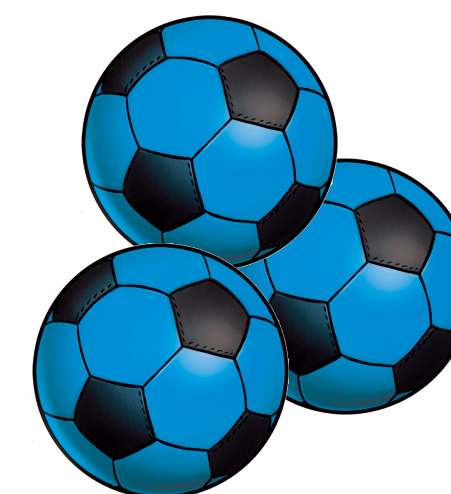
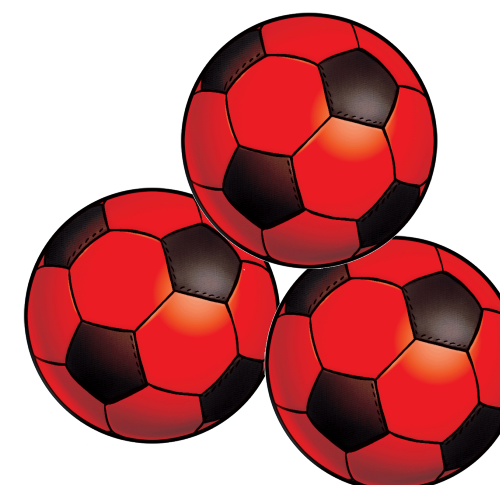
$$15 \div 3 = 5$$

"If I shared my 15 footballs fairly into 3 bags, how many balls would be in each bag?" "5"

Grouping

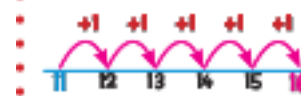
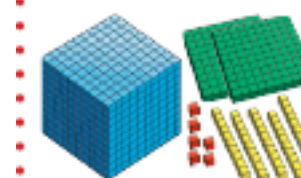
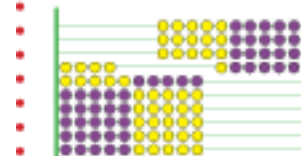
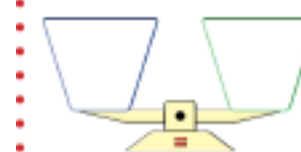
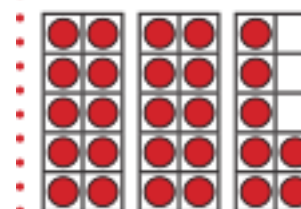
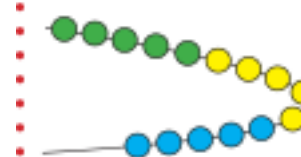


5 groups



$$15 \div 3 = 5$$

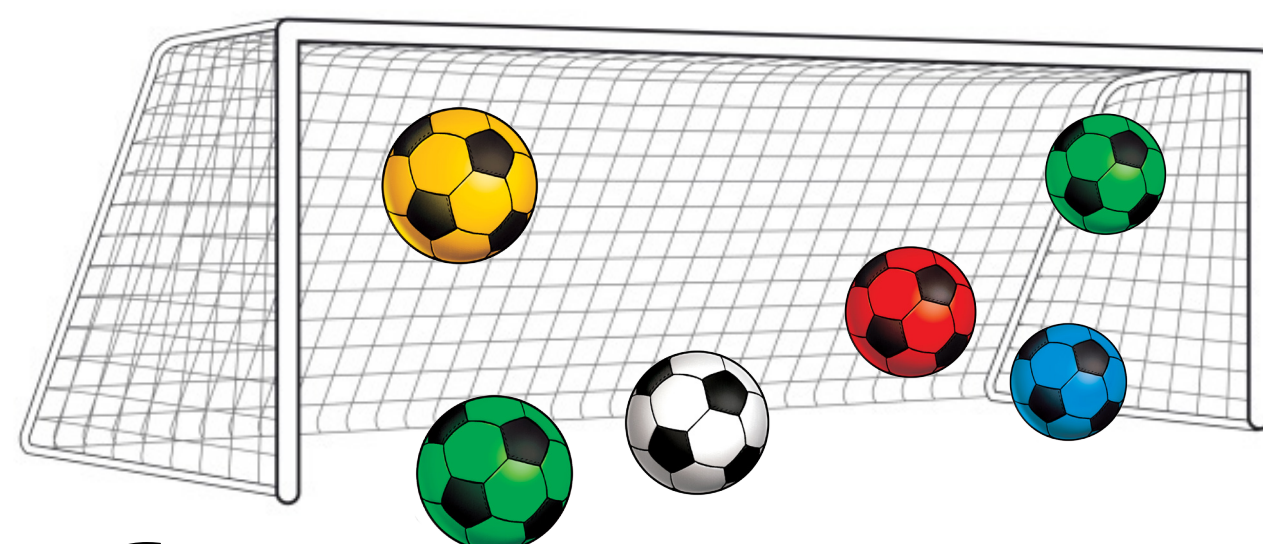
"If I can put my 15 footballs into groups of 3, how many groups would I create?" "5"



$$12 \div 2 = 6$$

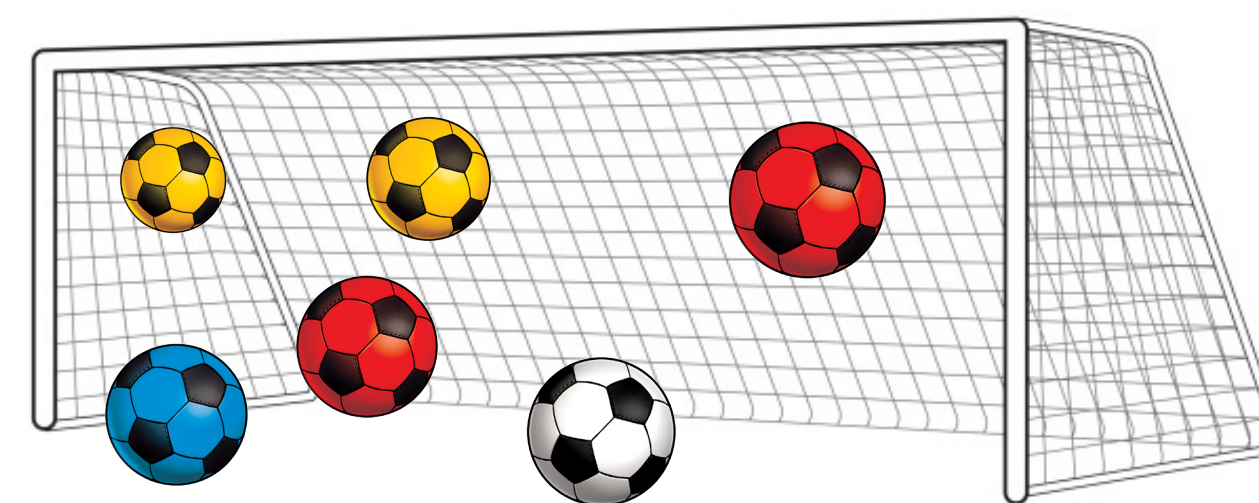


Sharing Model

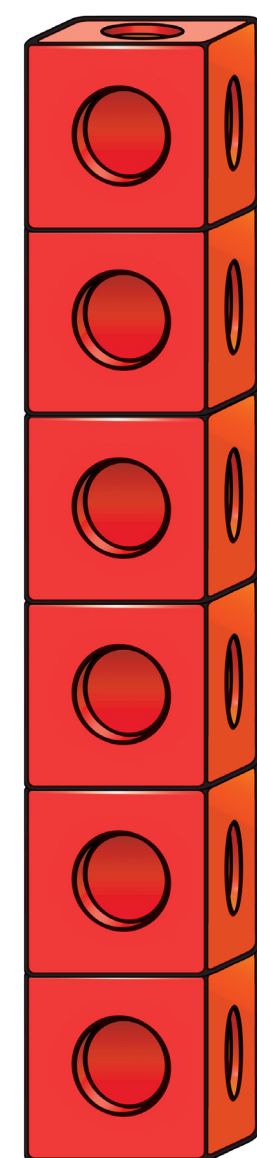


6

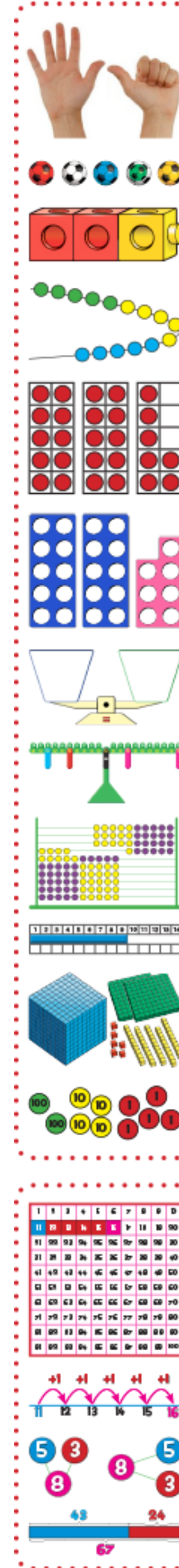
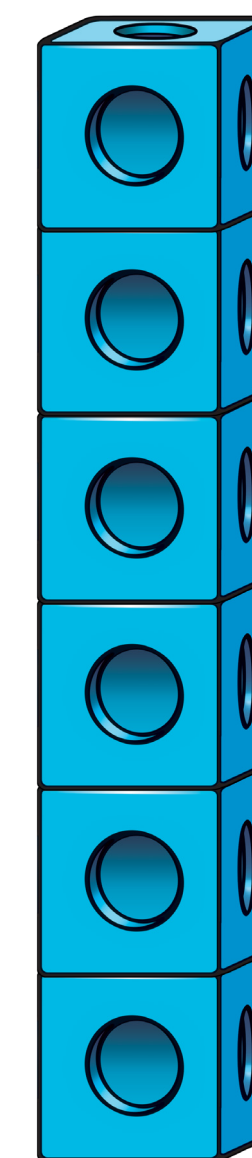
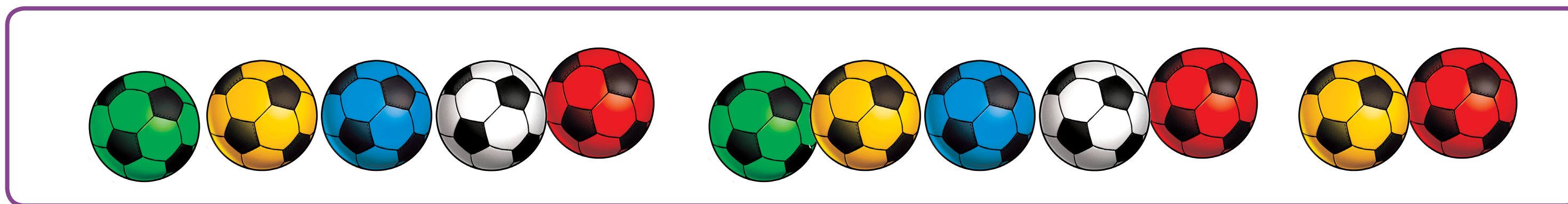
**6 footballs
in each goal!**



6



12



$$12 \div 2 = 6$$

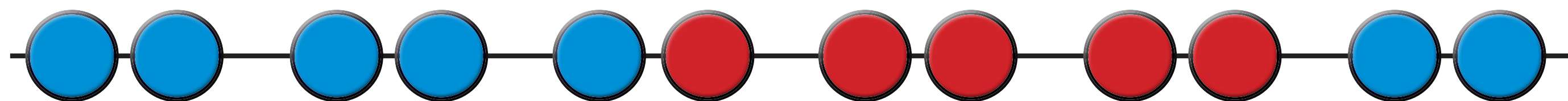
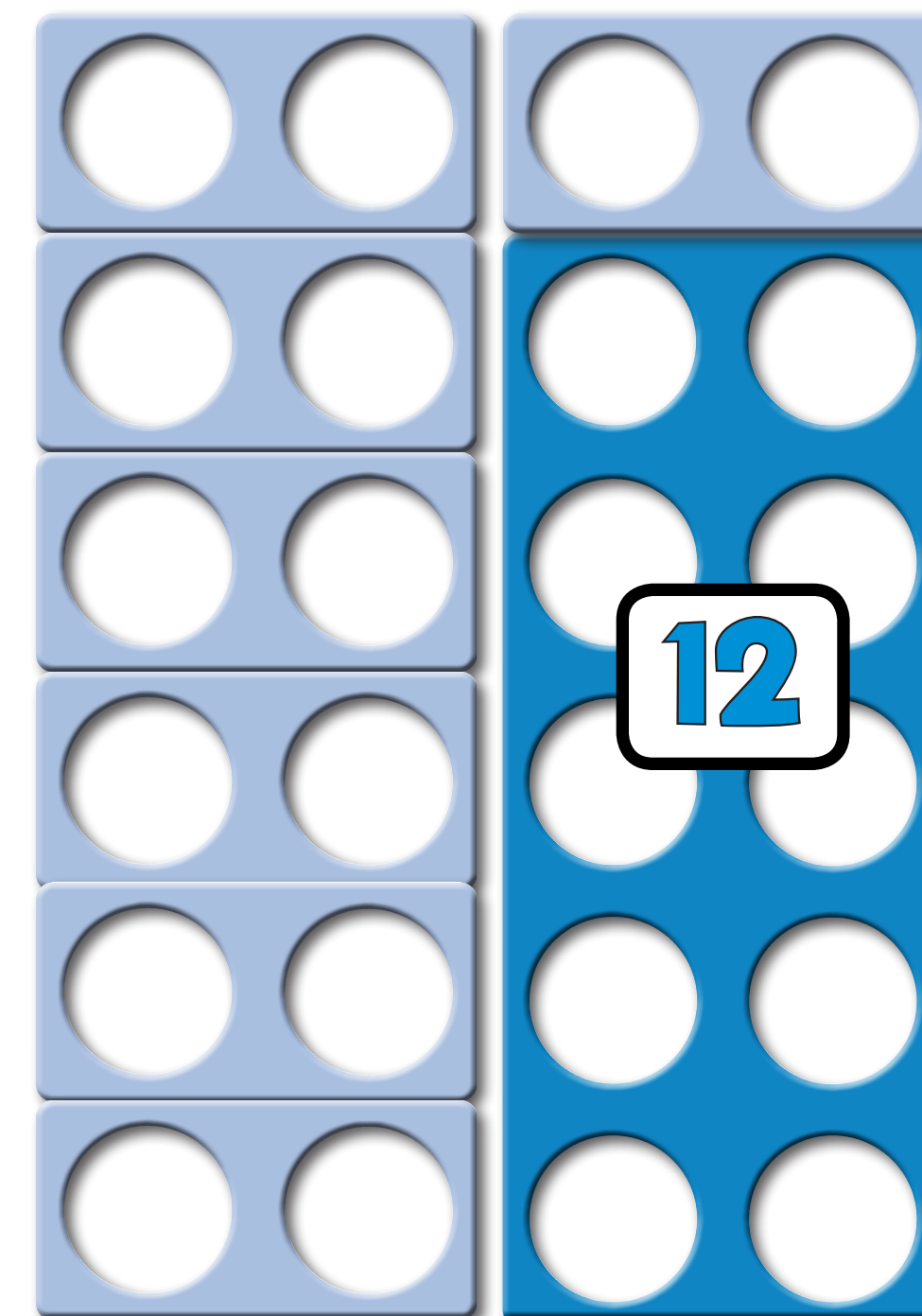
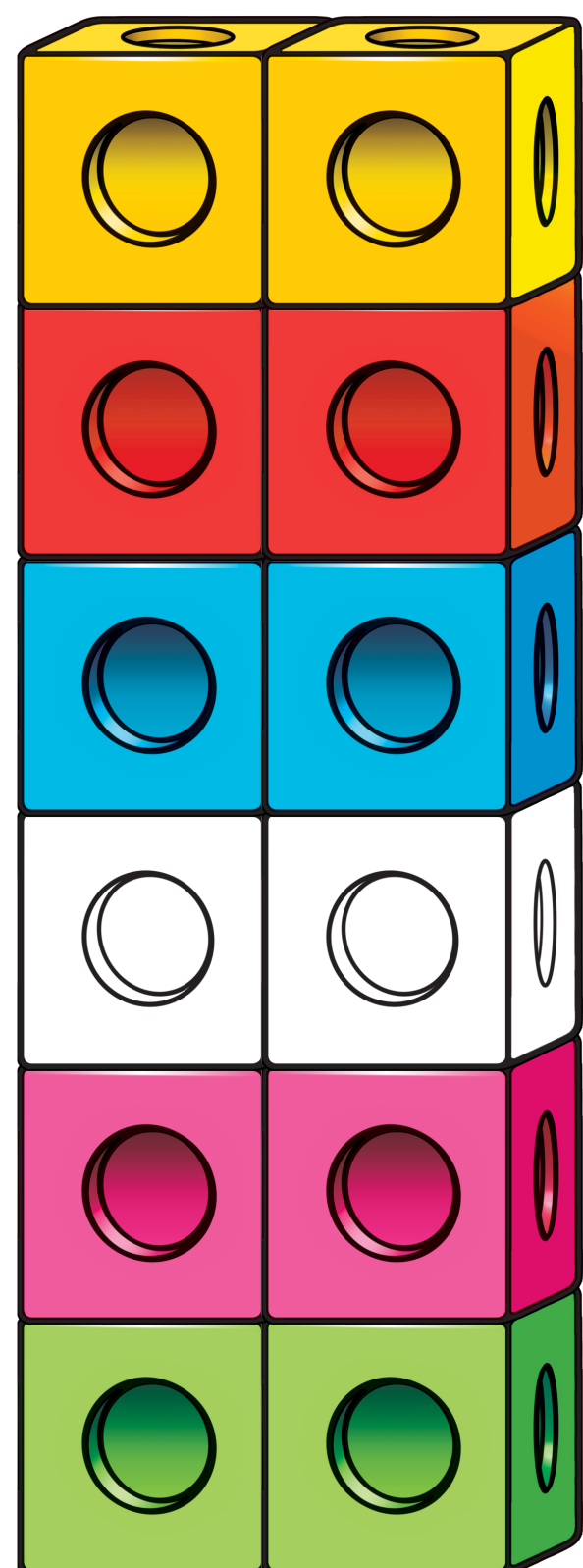


Grouping Model

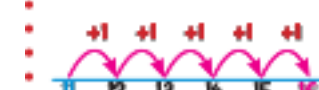
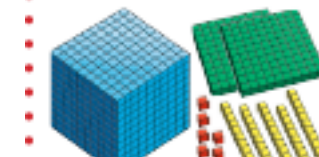
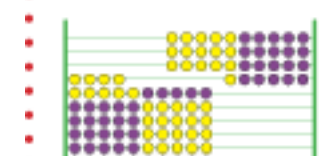
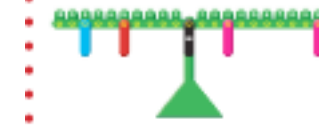
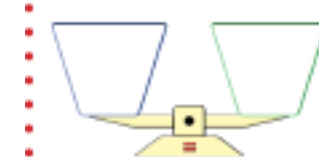
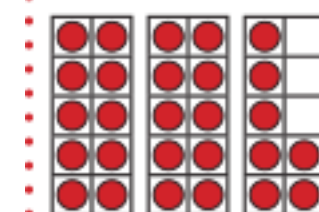
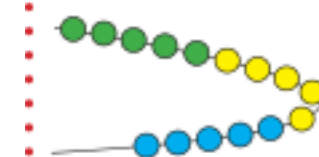
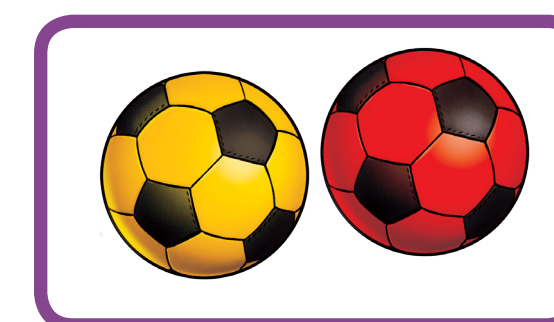
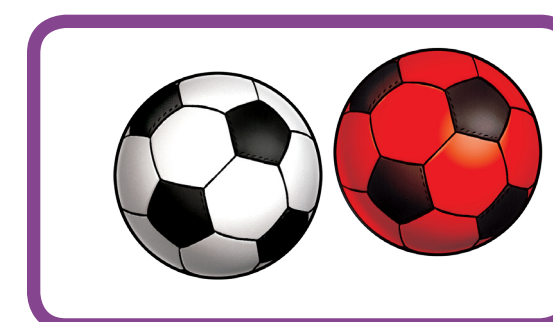
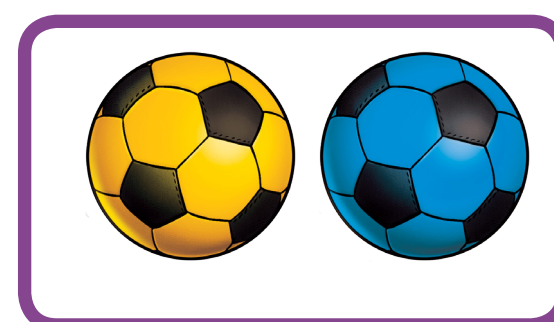
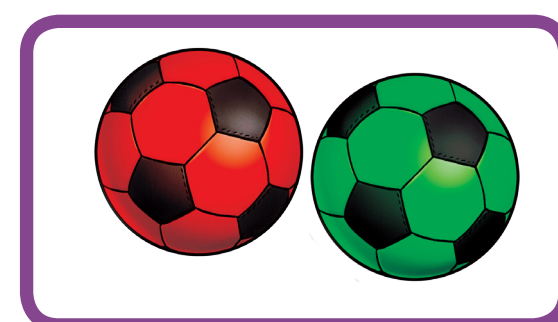
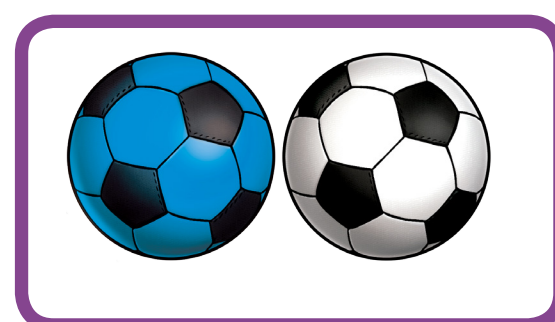
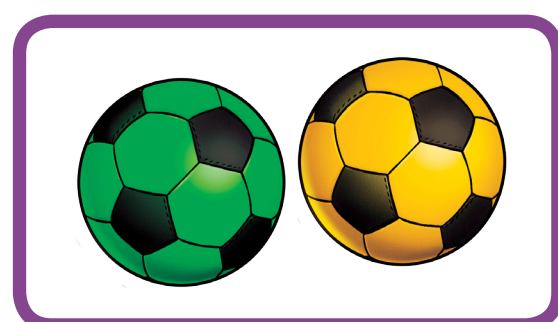


“12 divided by 2”

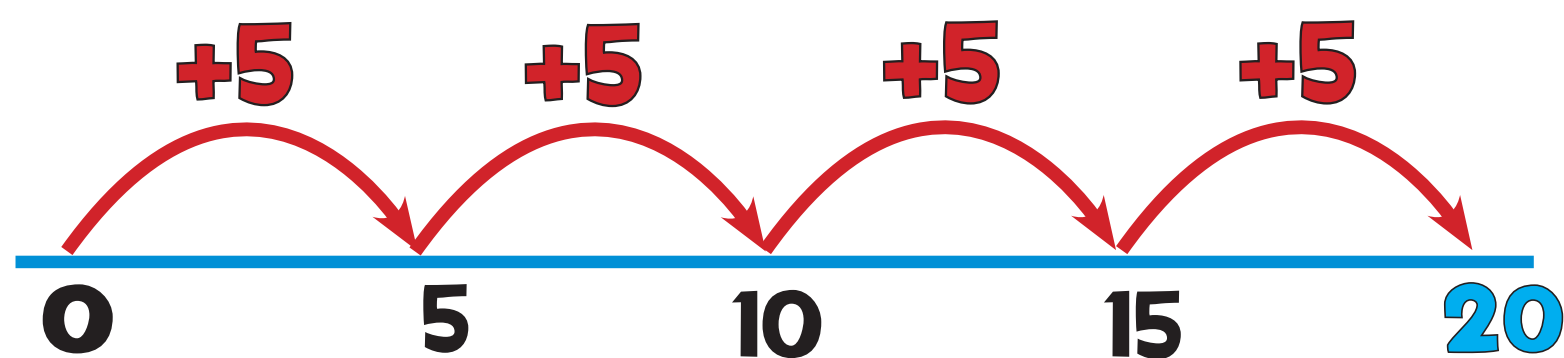
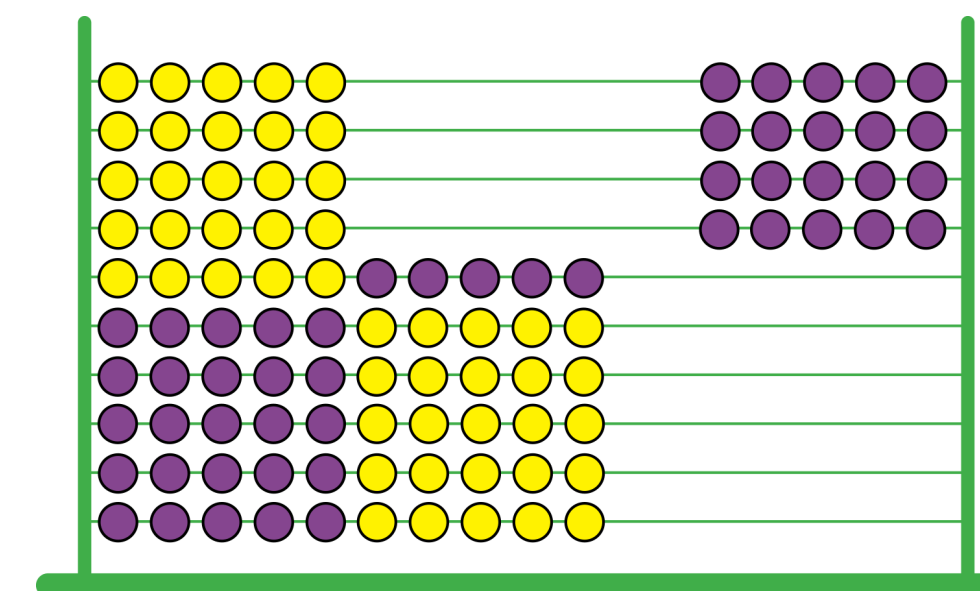
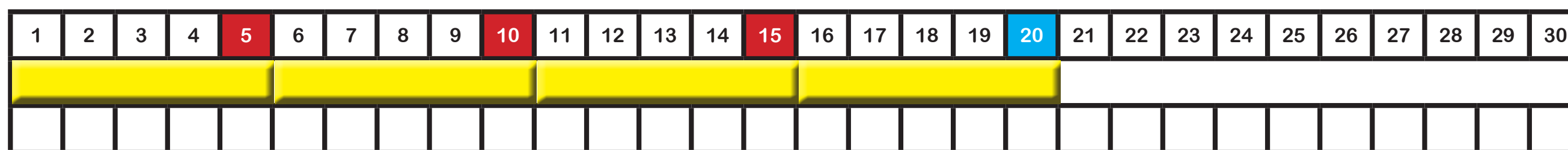
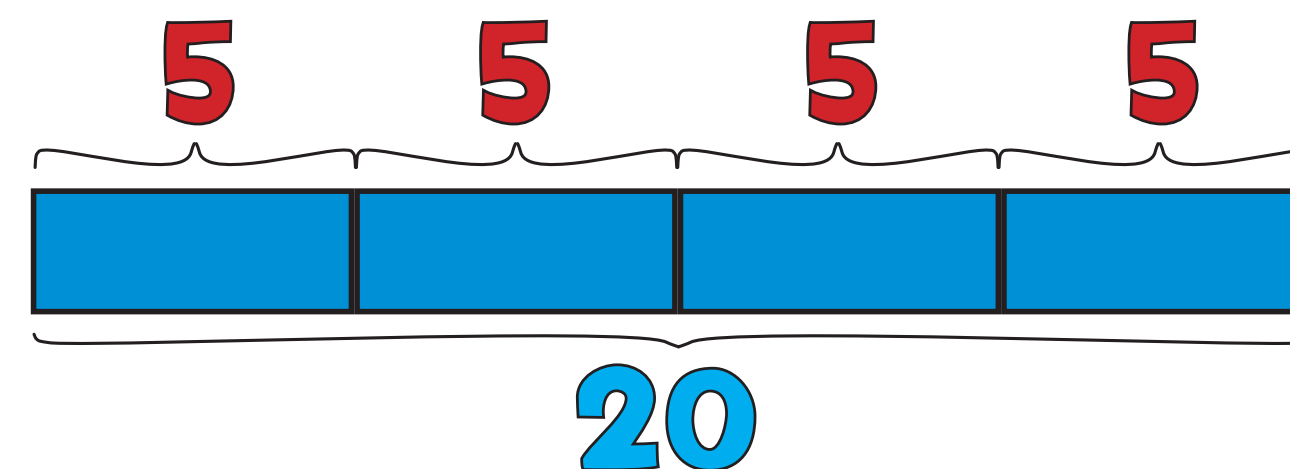
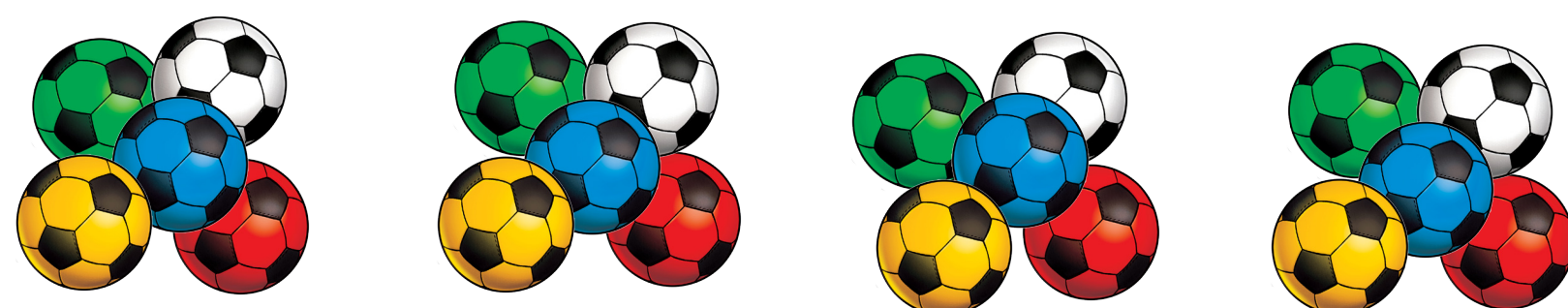
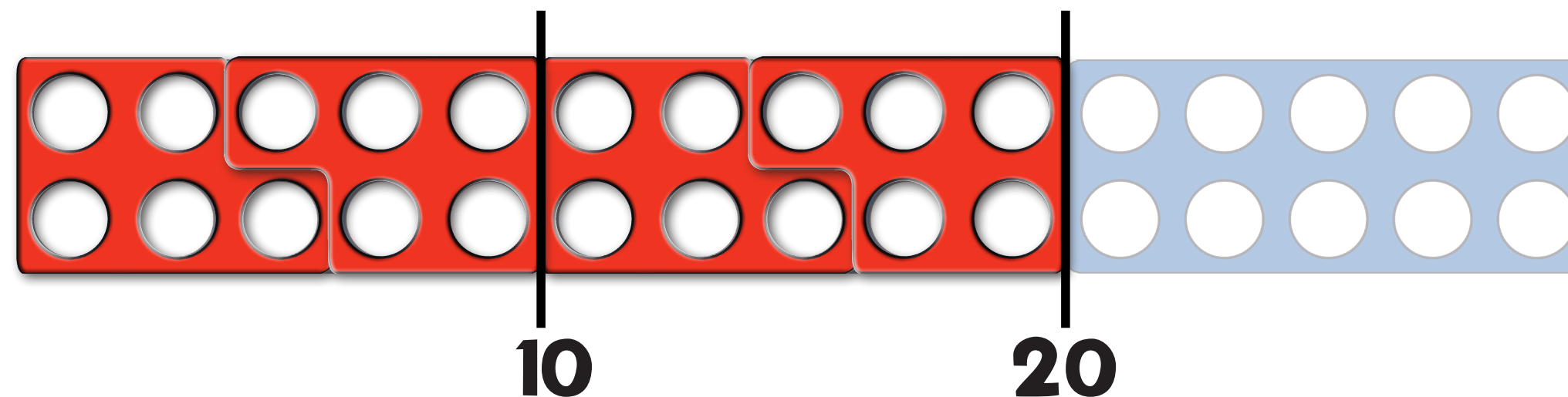
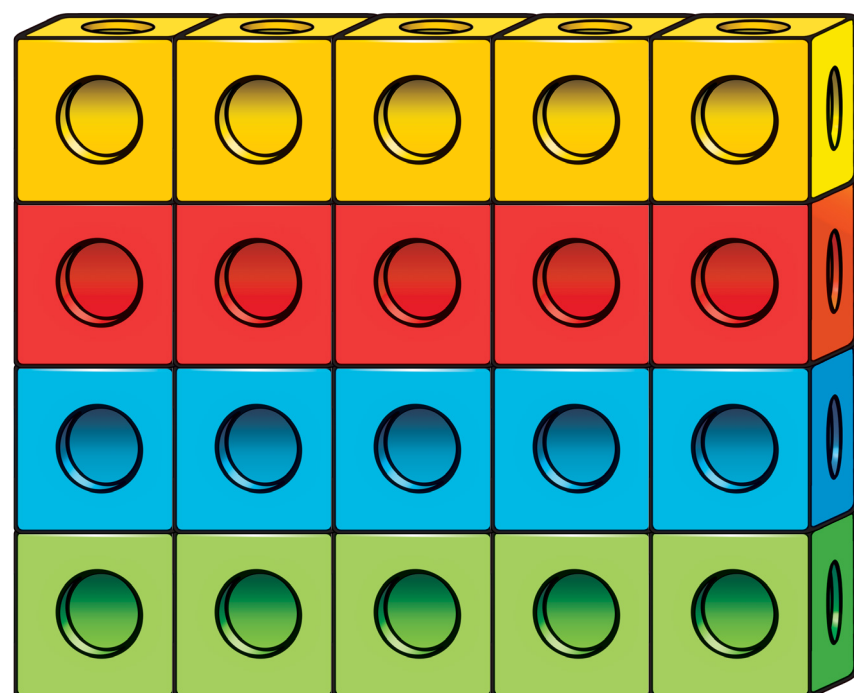
“How many 2s in 12?”



12 footballs in 6 groups of 2!

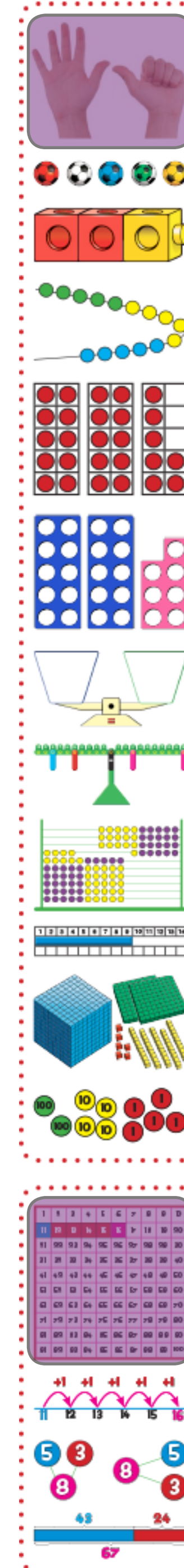
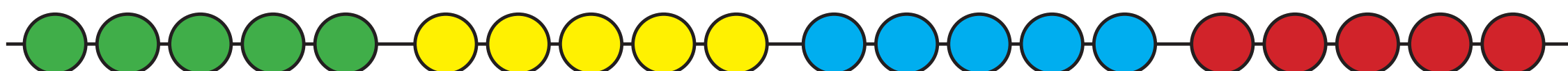


$$20 \div 5 = 4$$

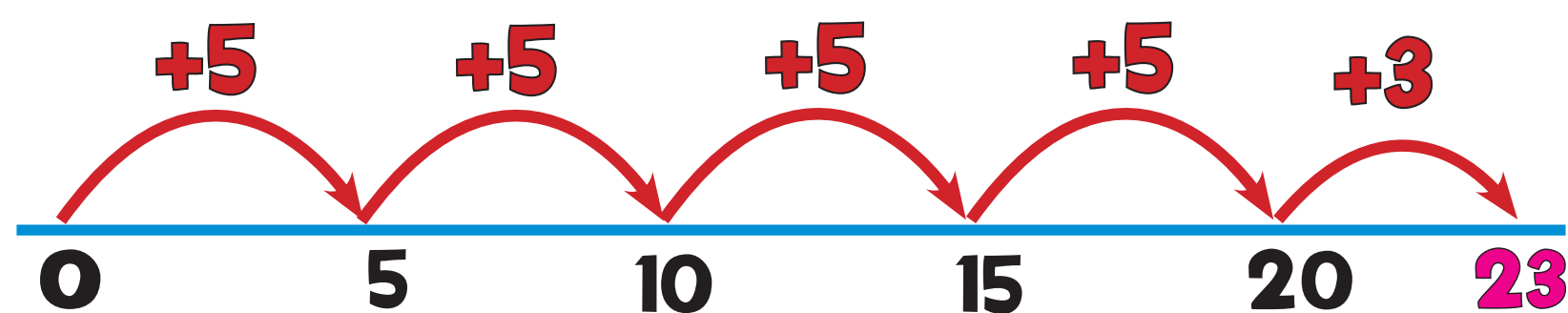
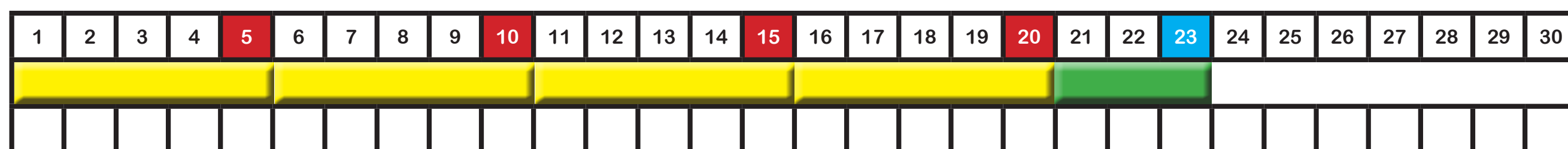
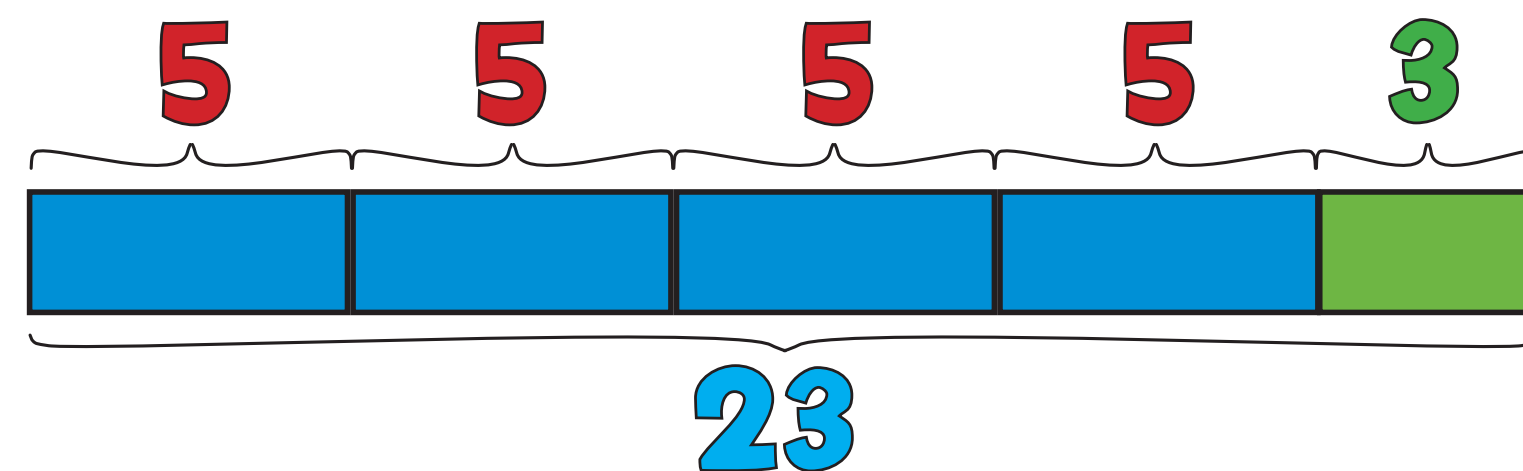
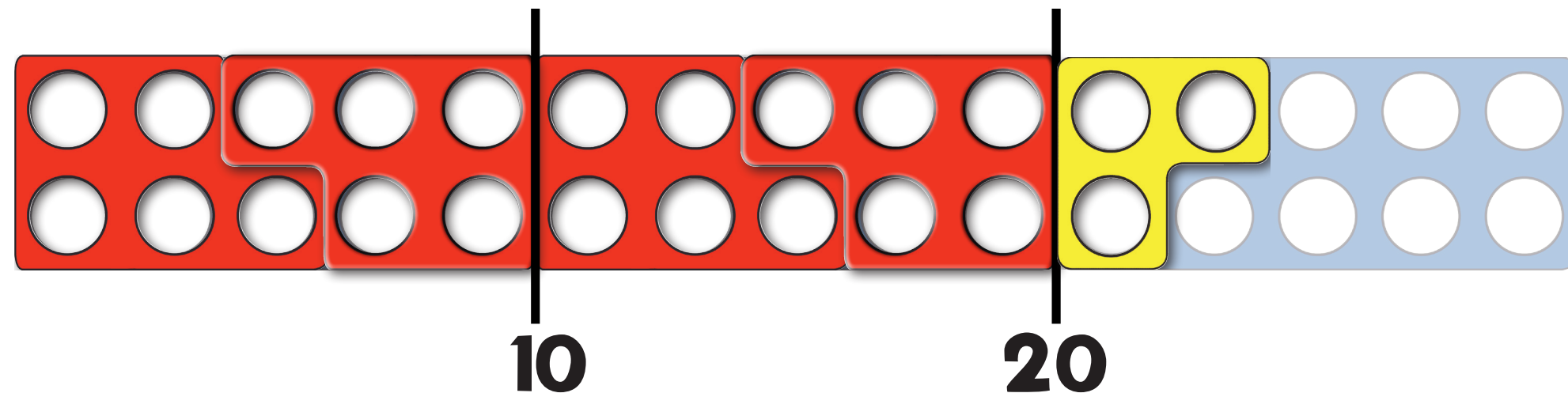
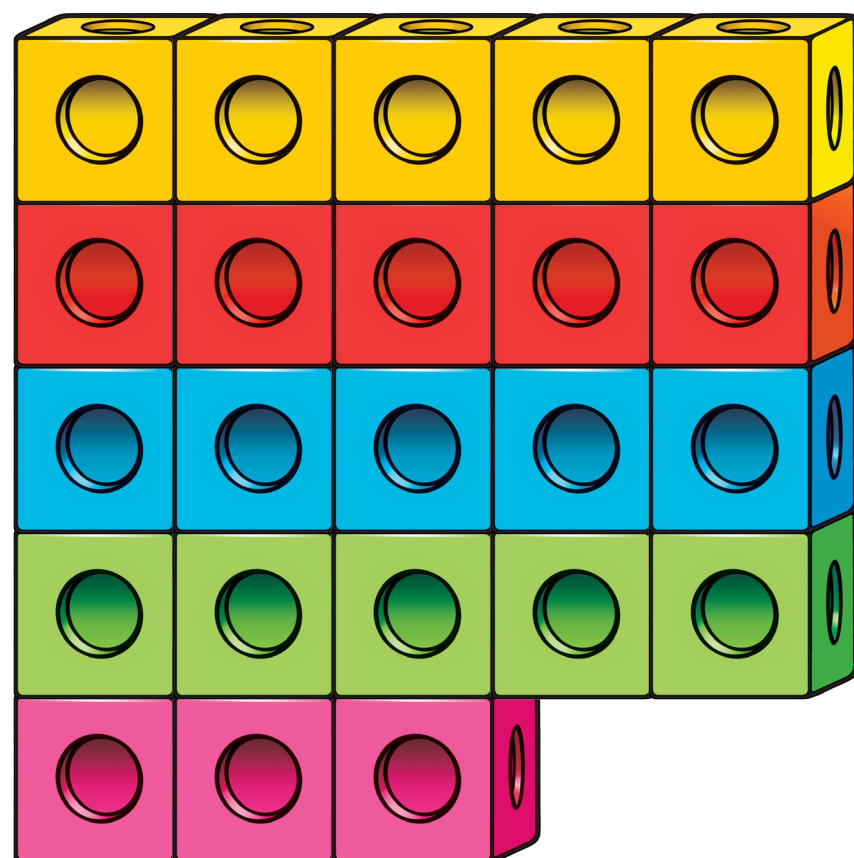


"How many 5s in 20?"

"20 divided by 5"

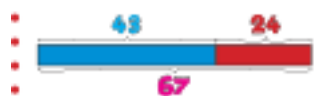
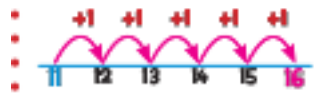
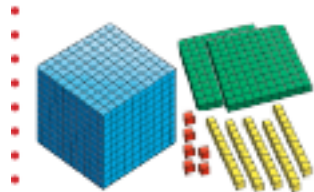
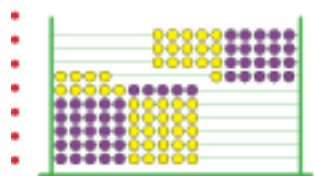
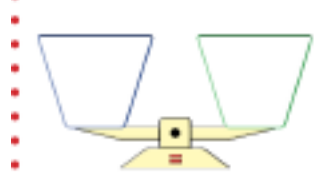
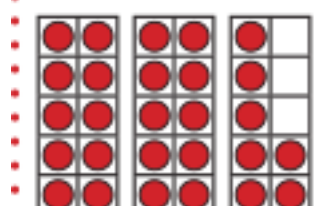
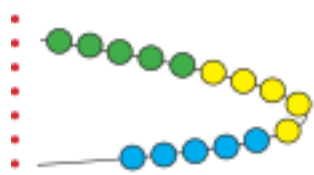
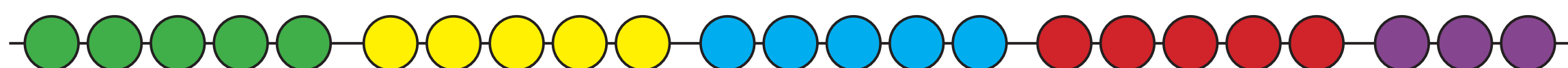


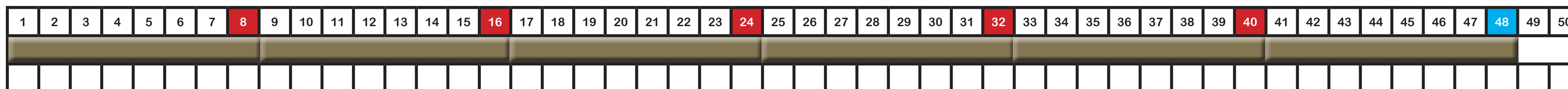
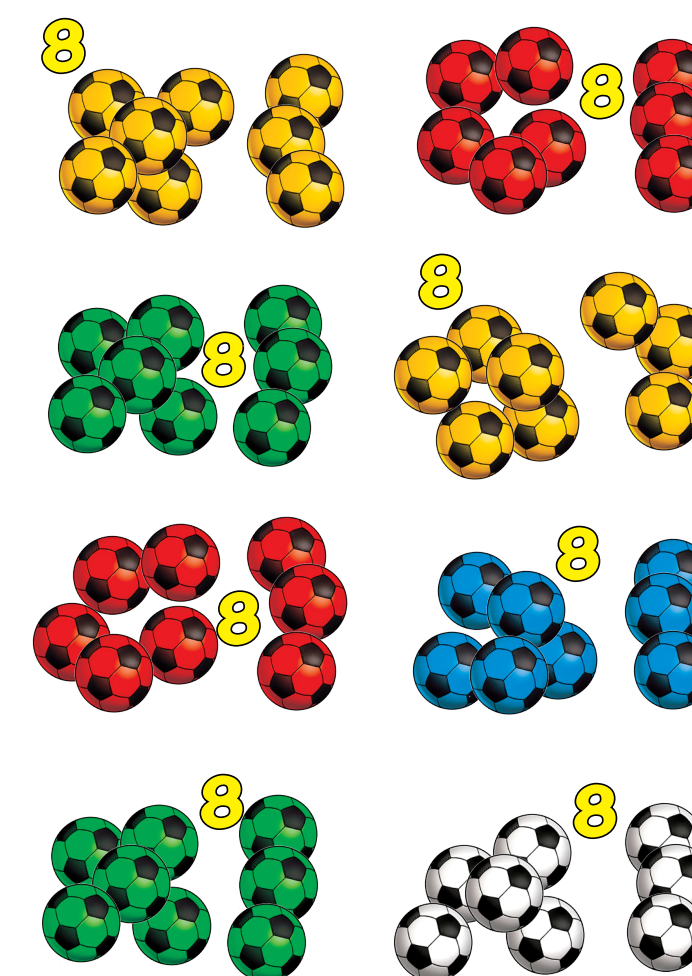
$$23 \div 5 = 4r3$$



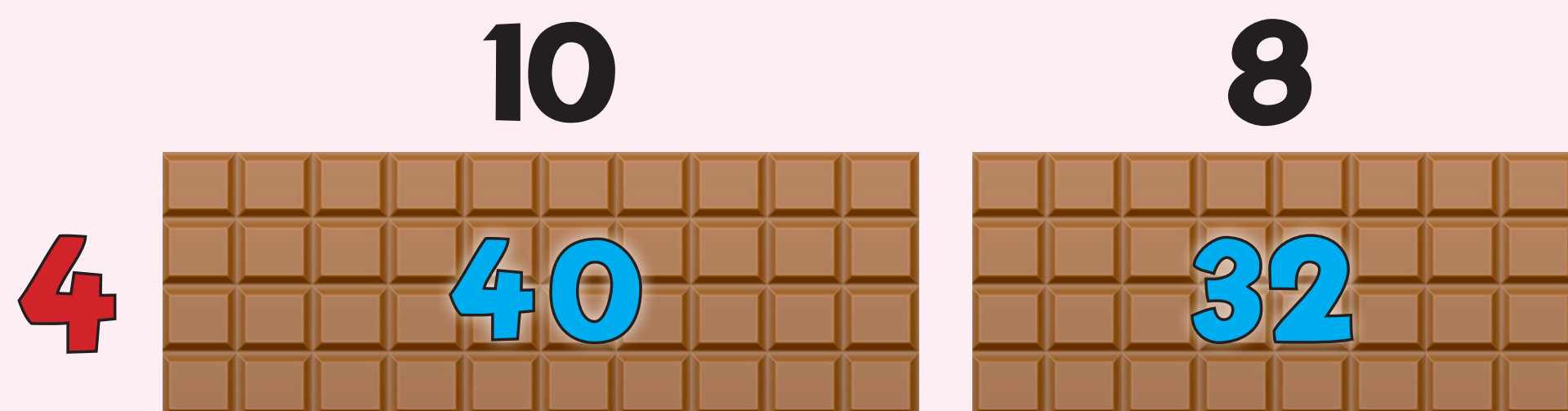
"How many 5s in 23?"

"23 divided by 5"





$$72 \div 4 = 18$$



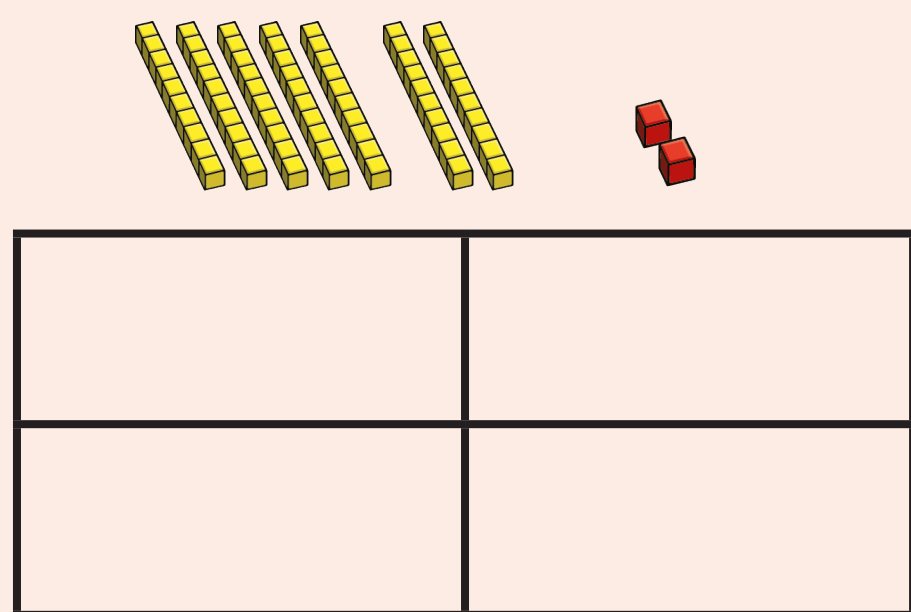
$$40 + 32 \div 4$$

$$10 + 8 = 18$$

Grouping Image!

1

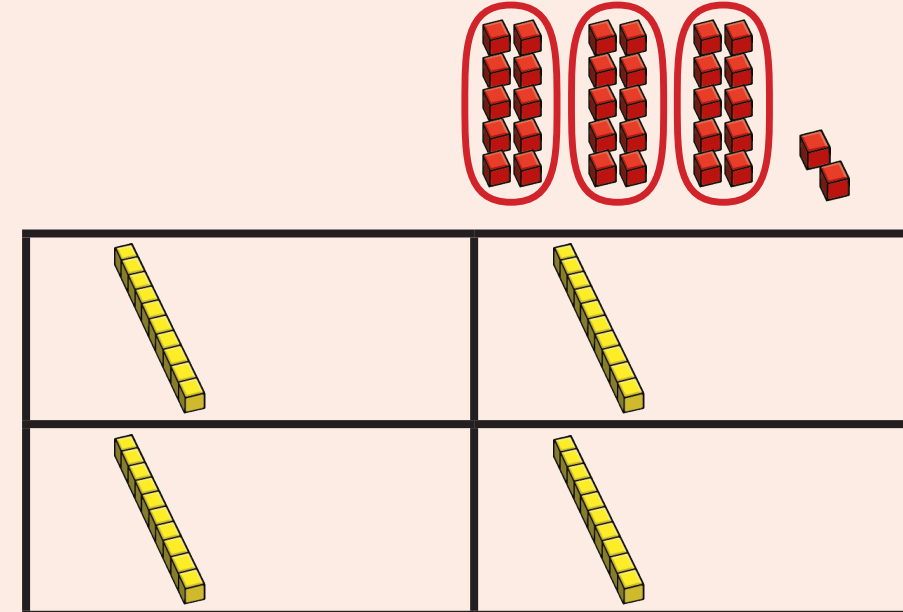
Tens **Ones**



$$4 \overline{) 72}$$

3

Tens **Ones**

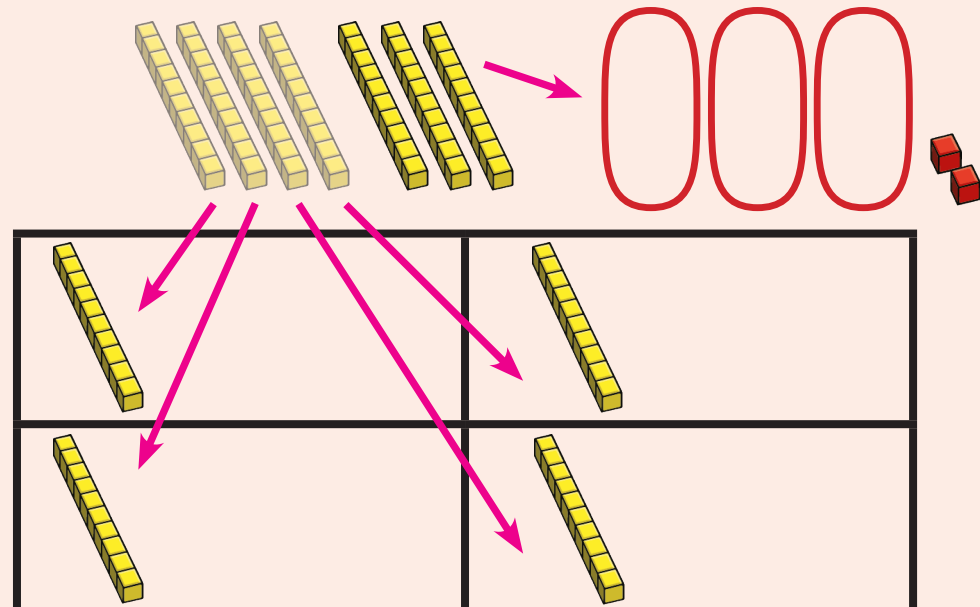


$$4 \overline{) 72} \begin{matrix} 1 \\ 8 \end{matrix}$$

Sharing Image!

2

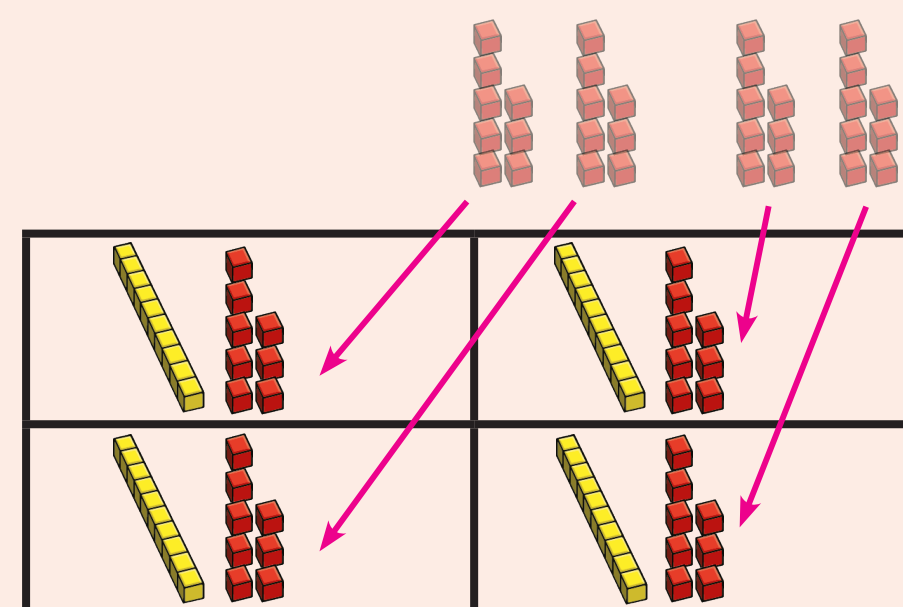
Tens **Ones**



$$4 \overline{) 72} \begin{matrix} 1 \\ 8 \end{matrix}$$

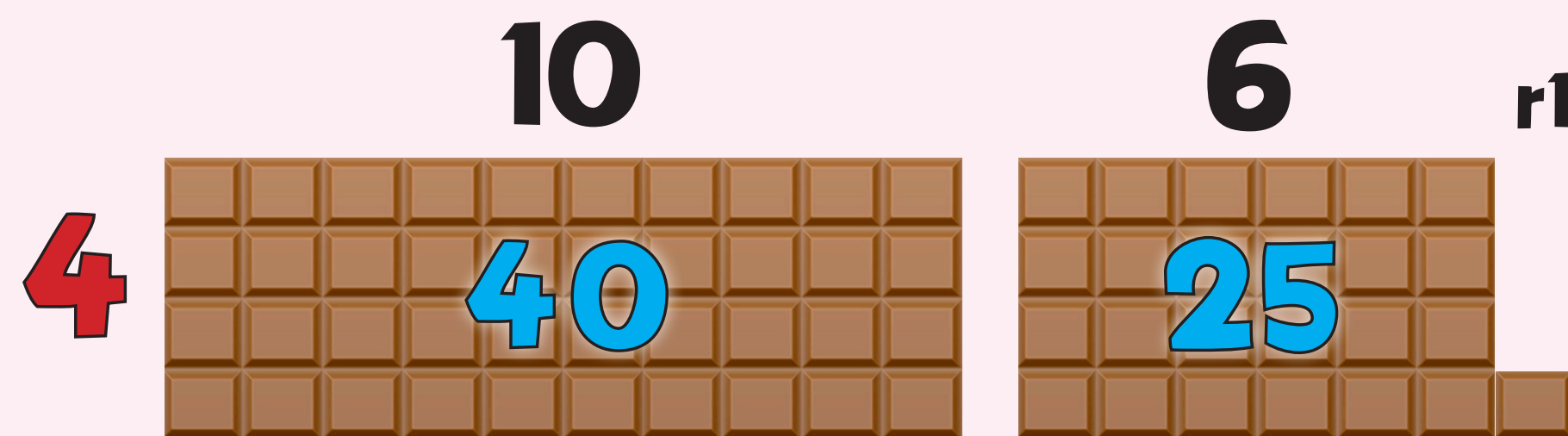
4

Tens **Ones**



$$4 \overline{) 72} \begin{matrix} 1 \\ 8 \end{matrix}$$

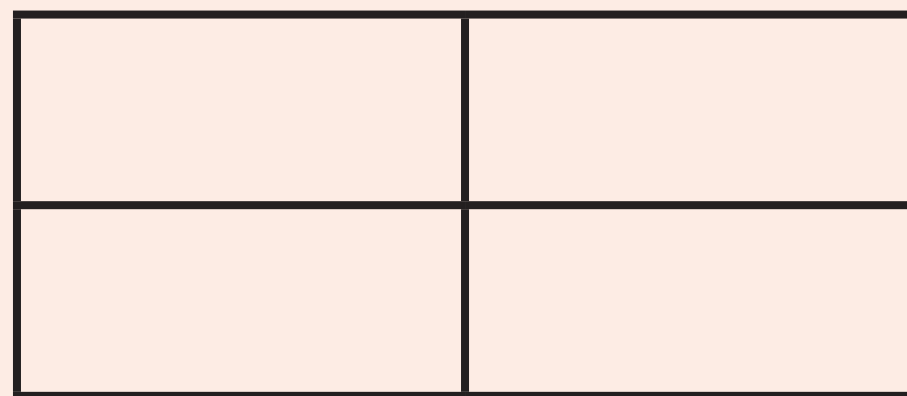
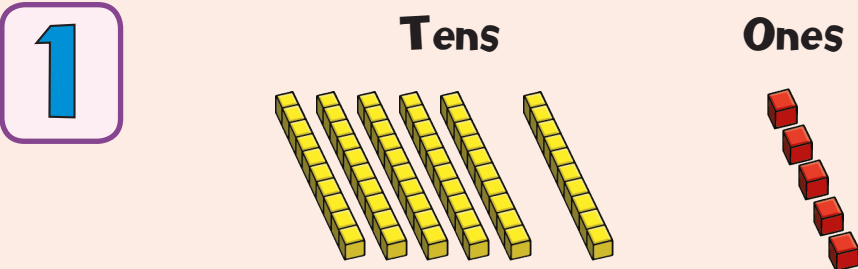
$$65 \div 4 = 16r1$$



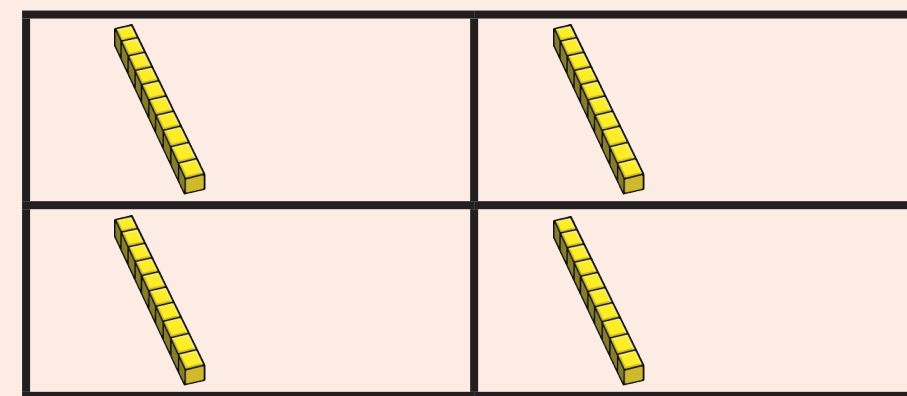
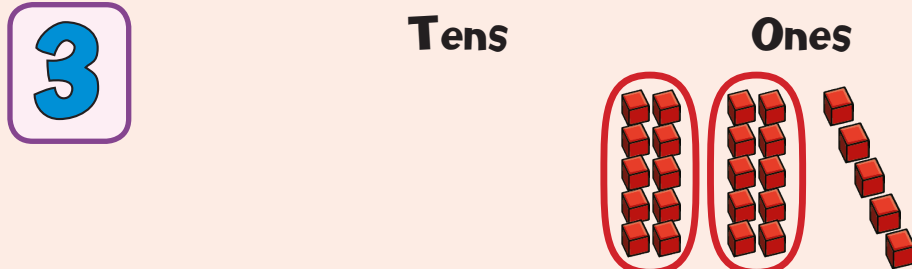
40 + **25** **Grouping Image!**

10 + **6r1** = **16r1**

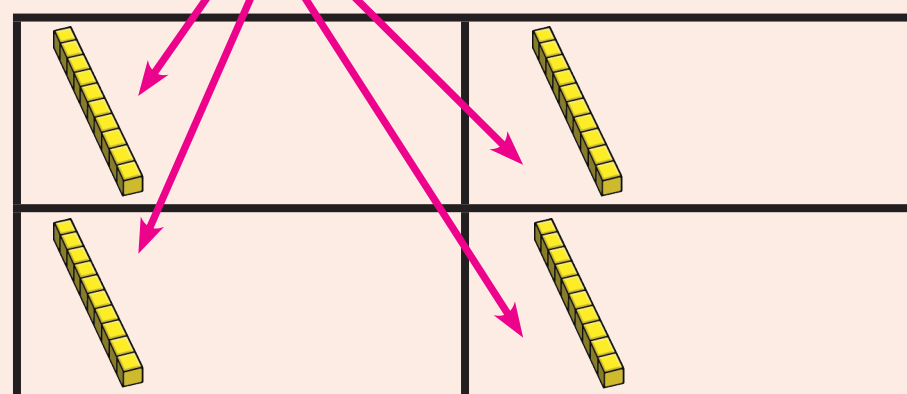
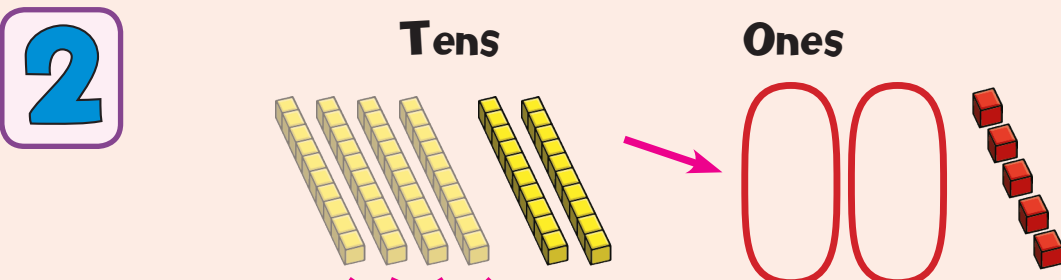
$\div 4$



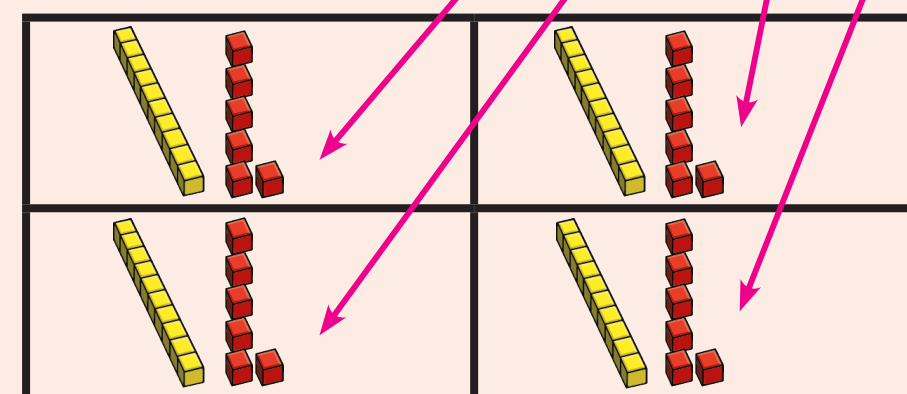
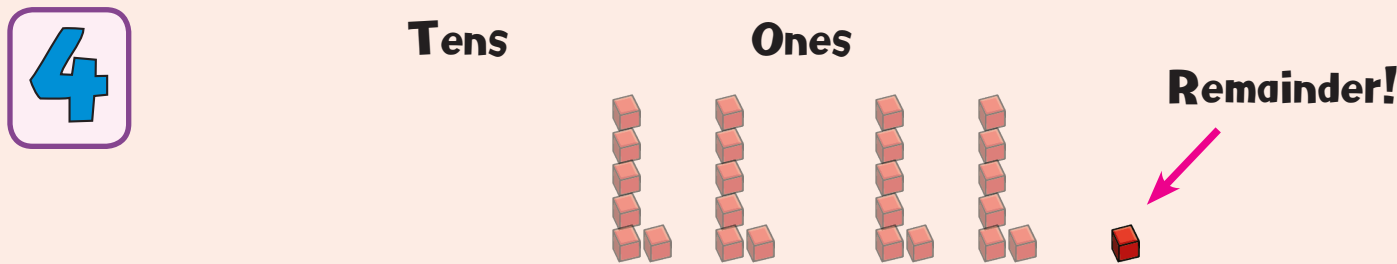
$$4 \overline{) 65}$$



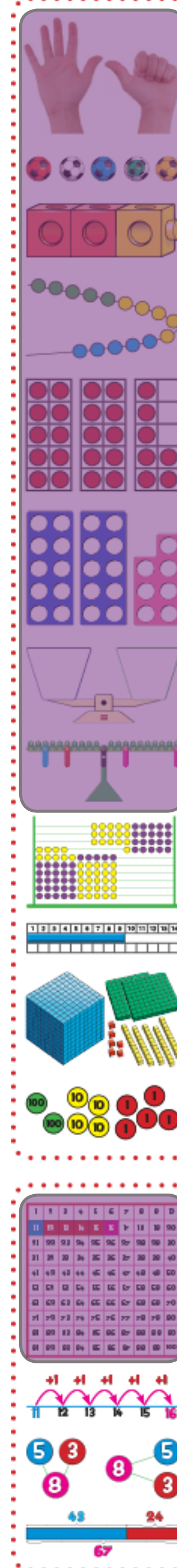
$$4 \overline{) 6^2 5}$$



$$4 \overline{) 65}$$



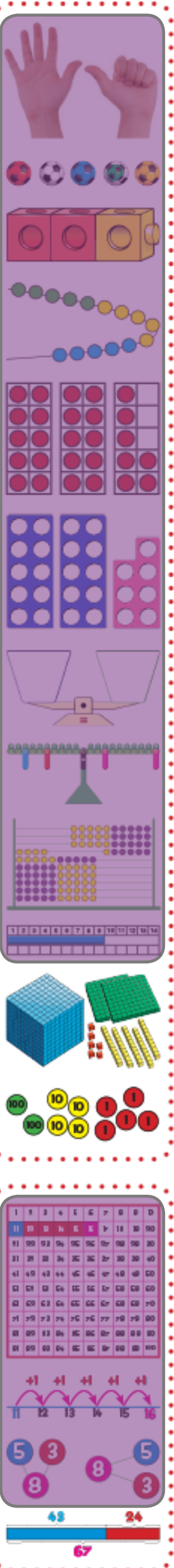
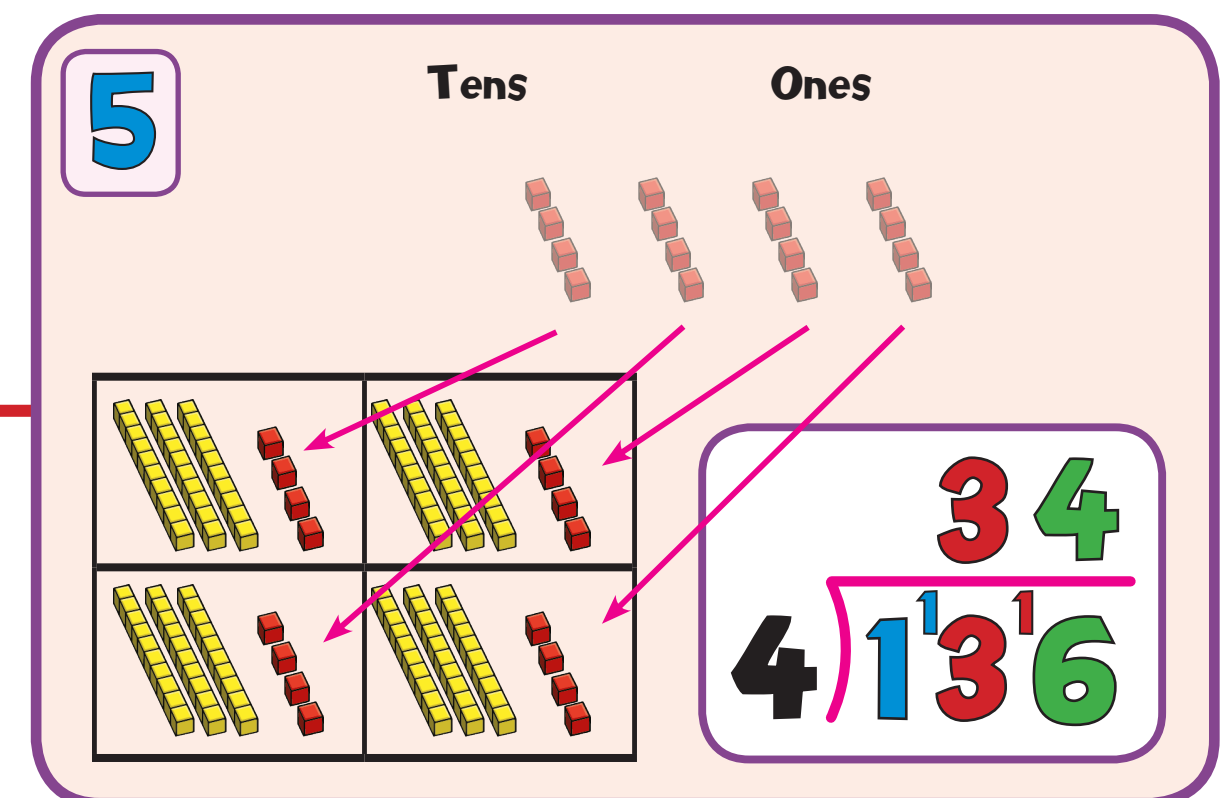
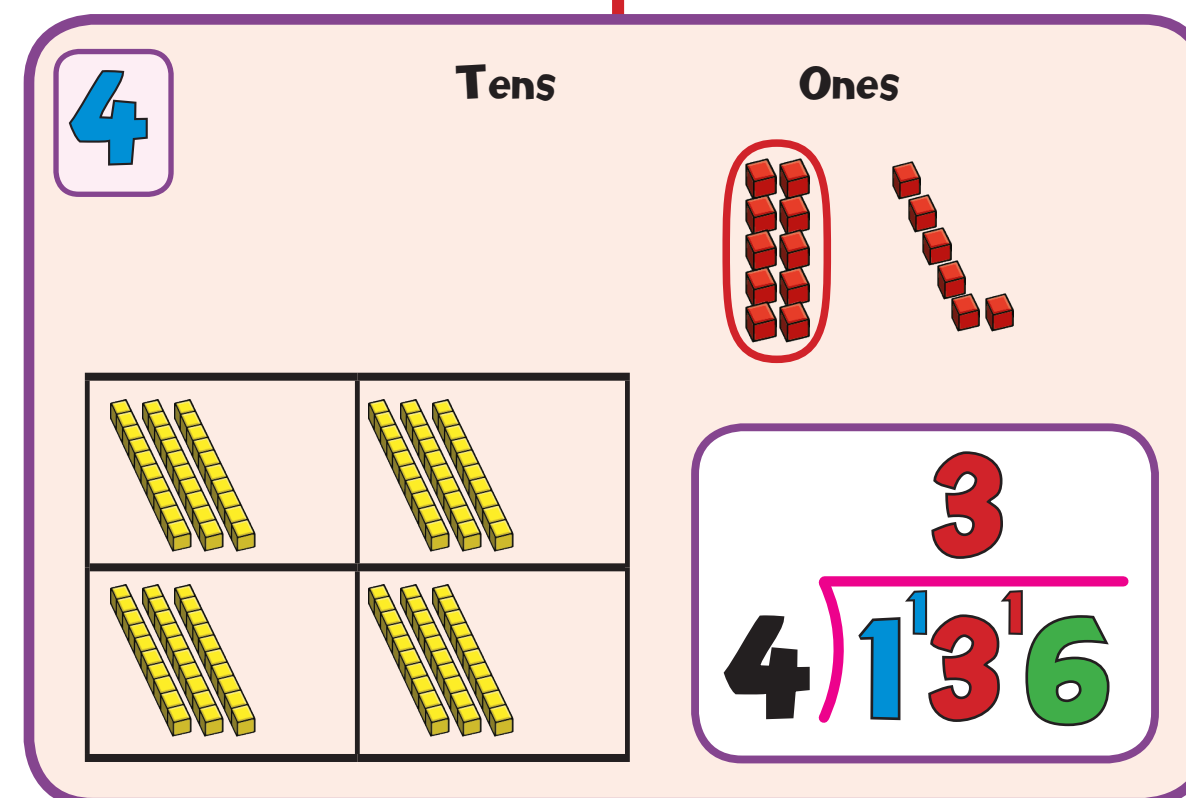
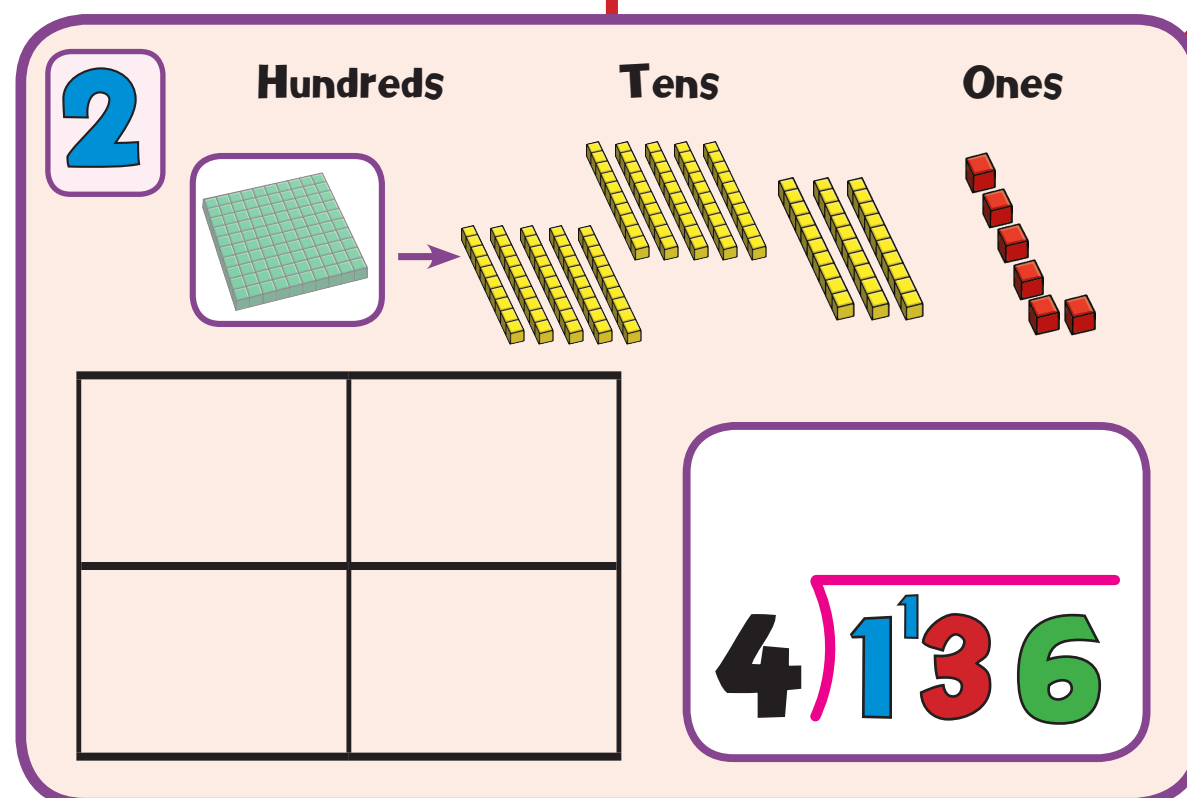
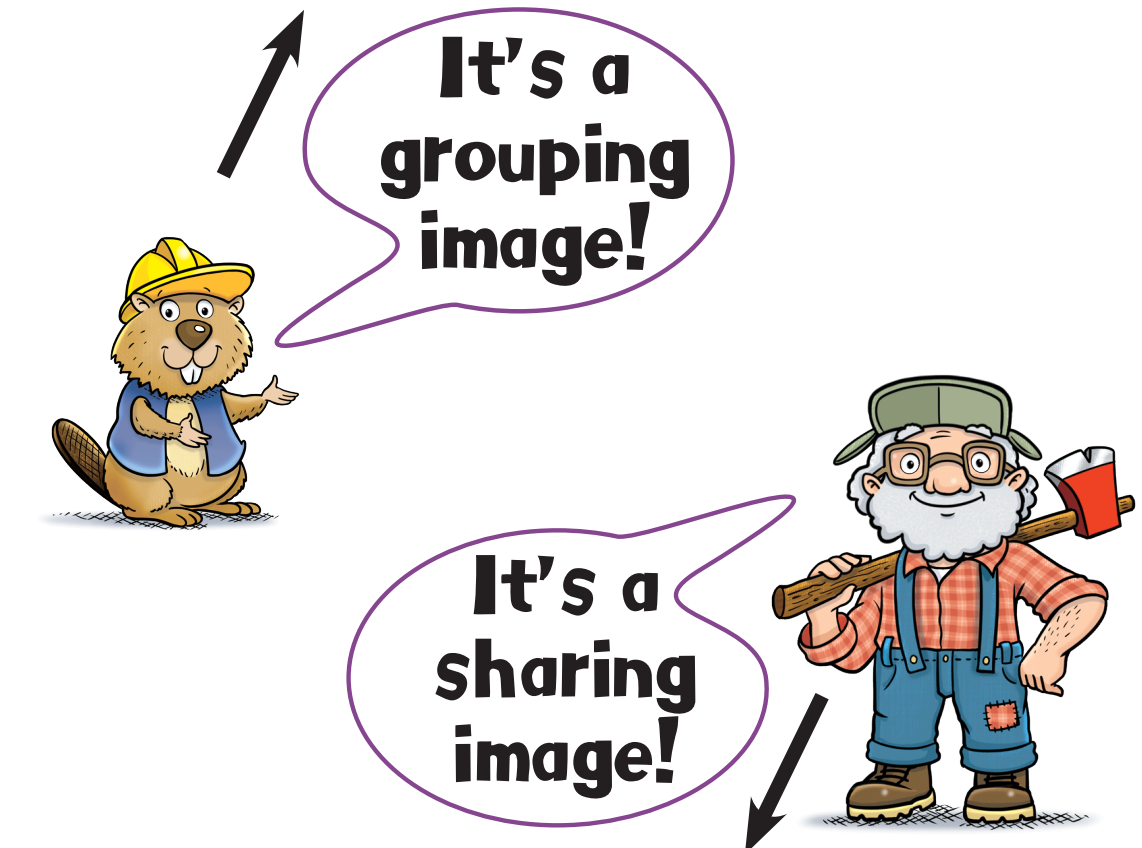
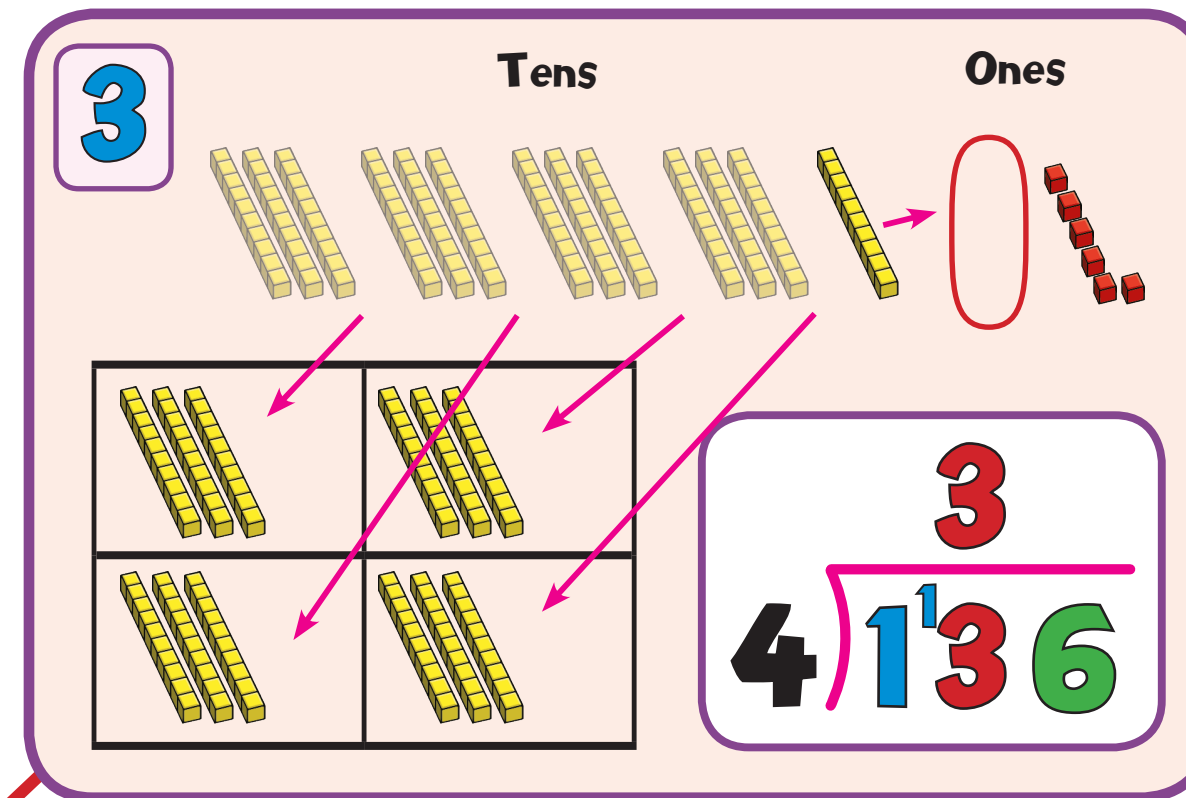
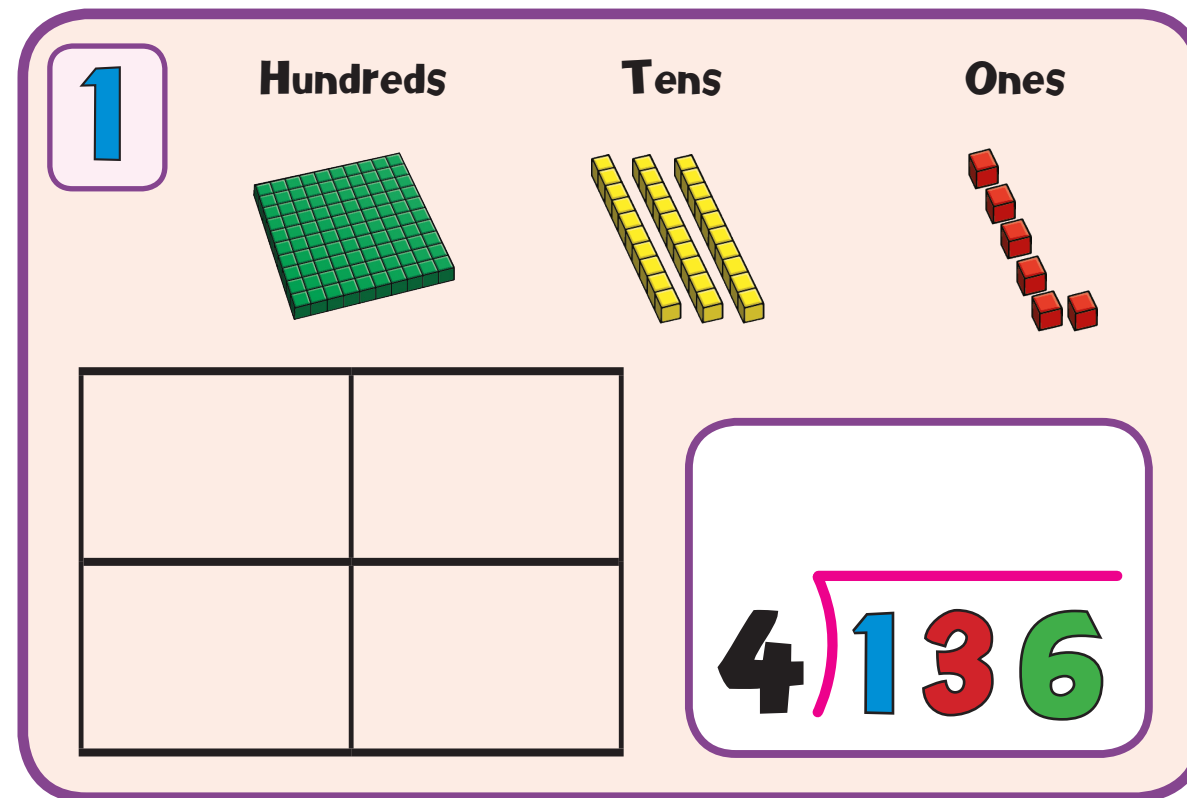
$$4 \overline{) 65} 16r1$$



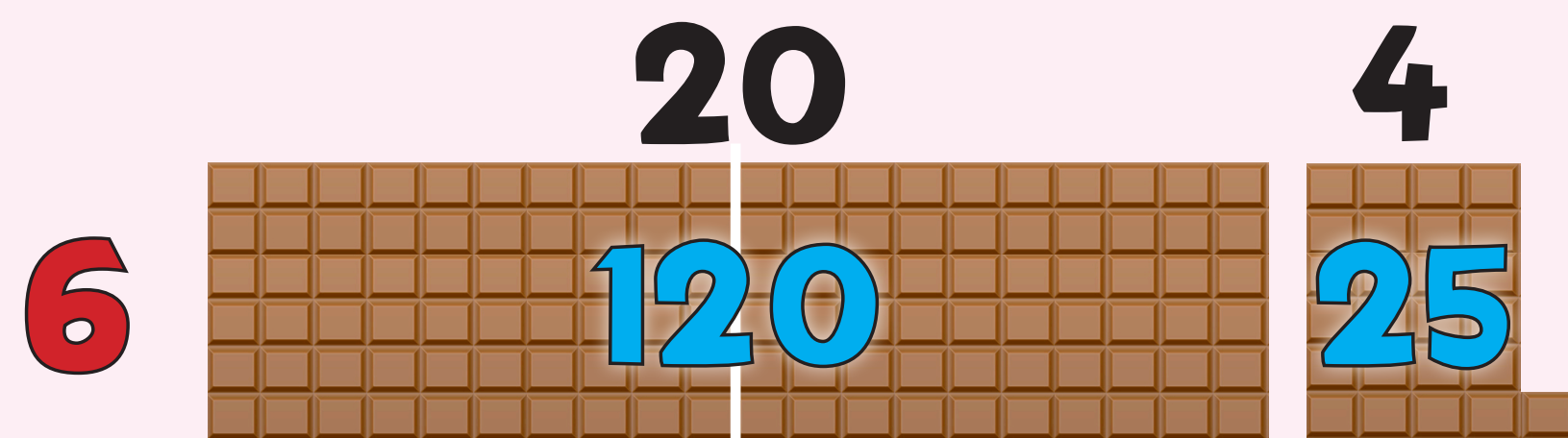
$$136 \div 4 = 34$$



$$\begin{array}{r} 120 \\ \downarrow \\ 30 \end{array} + \begin{array}{r} 16 \\ \downarrow \\ 4 \end{array} \div 4 = 34$$



$$145 \div 6 = 24 \text{ r}1$$



$$120 \div 6 = 20$$

$$25 \div 6 = 4 \text{ r}1$$

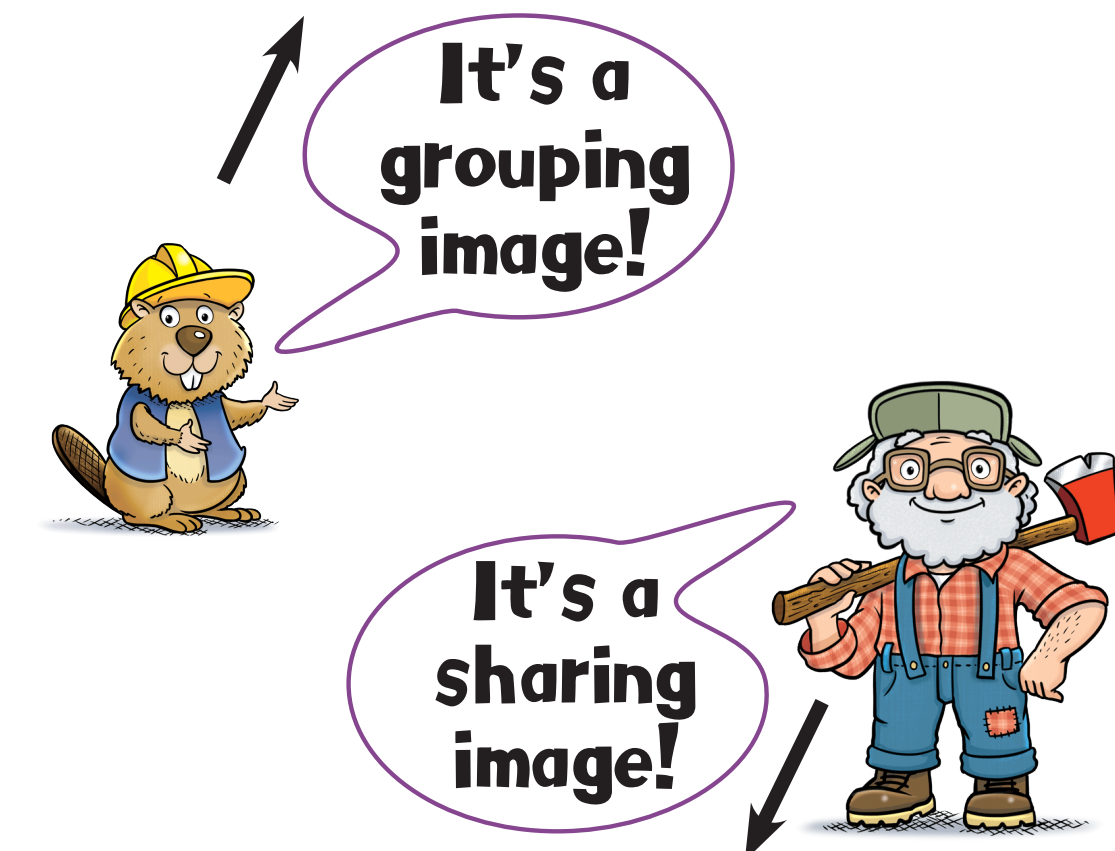
$$20 + 4 \text{ r}1 = 24 \text{ r}1$$

1 Hundreds Tens Ones

6 $\overline{)145}$

3 Tens Ones

2
6 $\overline{)14^25}$



2 Hundreds Tens Ones

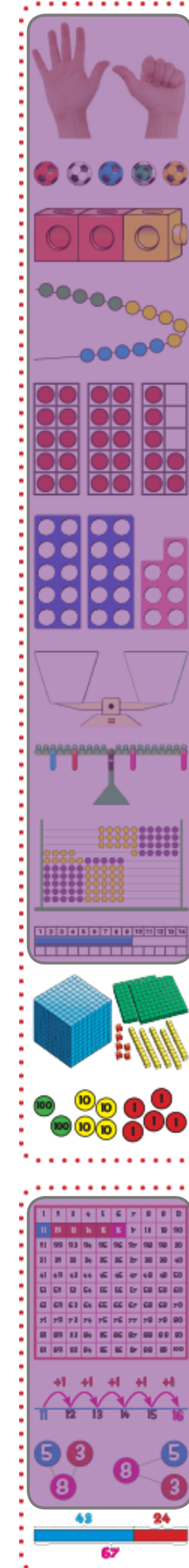
6 $\overline{)14^25}$

4 Tens Ones

2
6 $\overline{)14^25}$

5 Ones Remainder!

24 r1
6 $\overline{)14^25}$



$$394 \div 6 = 65r4$$

1

Hundreds Tens Ones

$6 \overline{) 394}$

2

Hundreds Tens Ones

$0 $
 $6 \overline{) 3^3 9^3 4}$

3

Tens Ones

$06 $
 $6 \overline{) 3^3 9^3 4}$

4

Ones

Sharing Image!

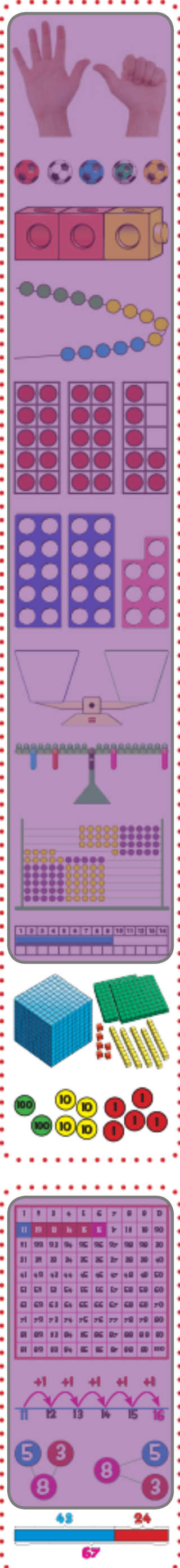
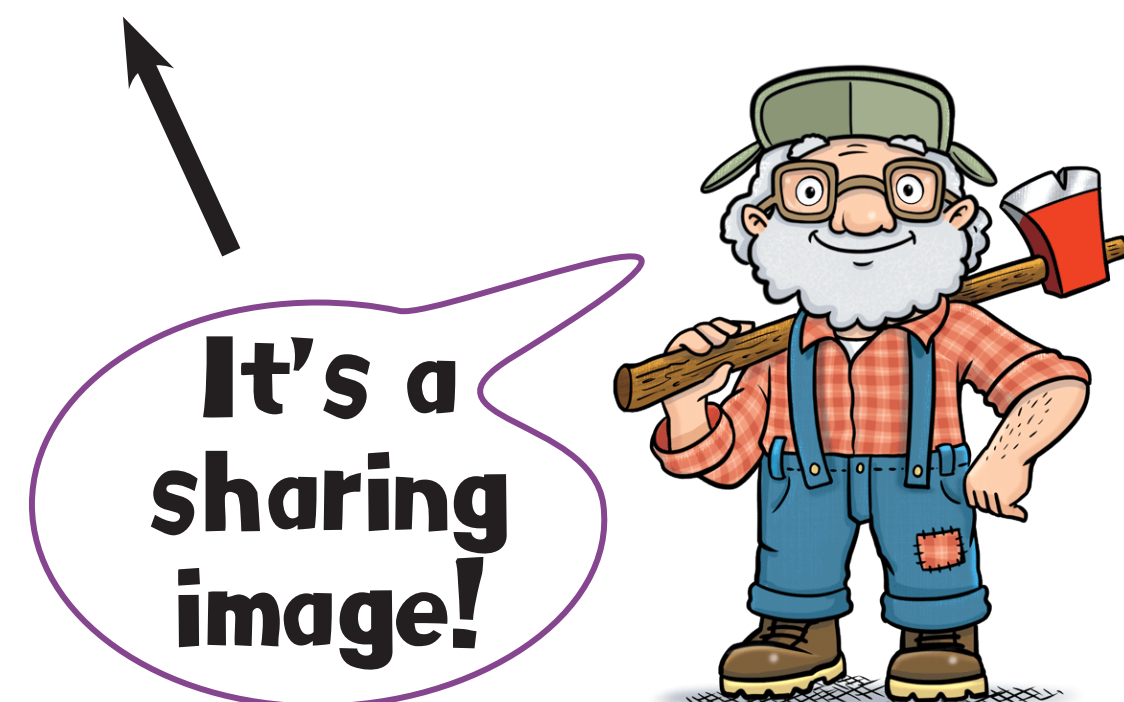
$06 $
 $6 \overline{) 3^3 9^3 4}$

5

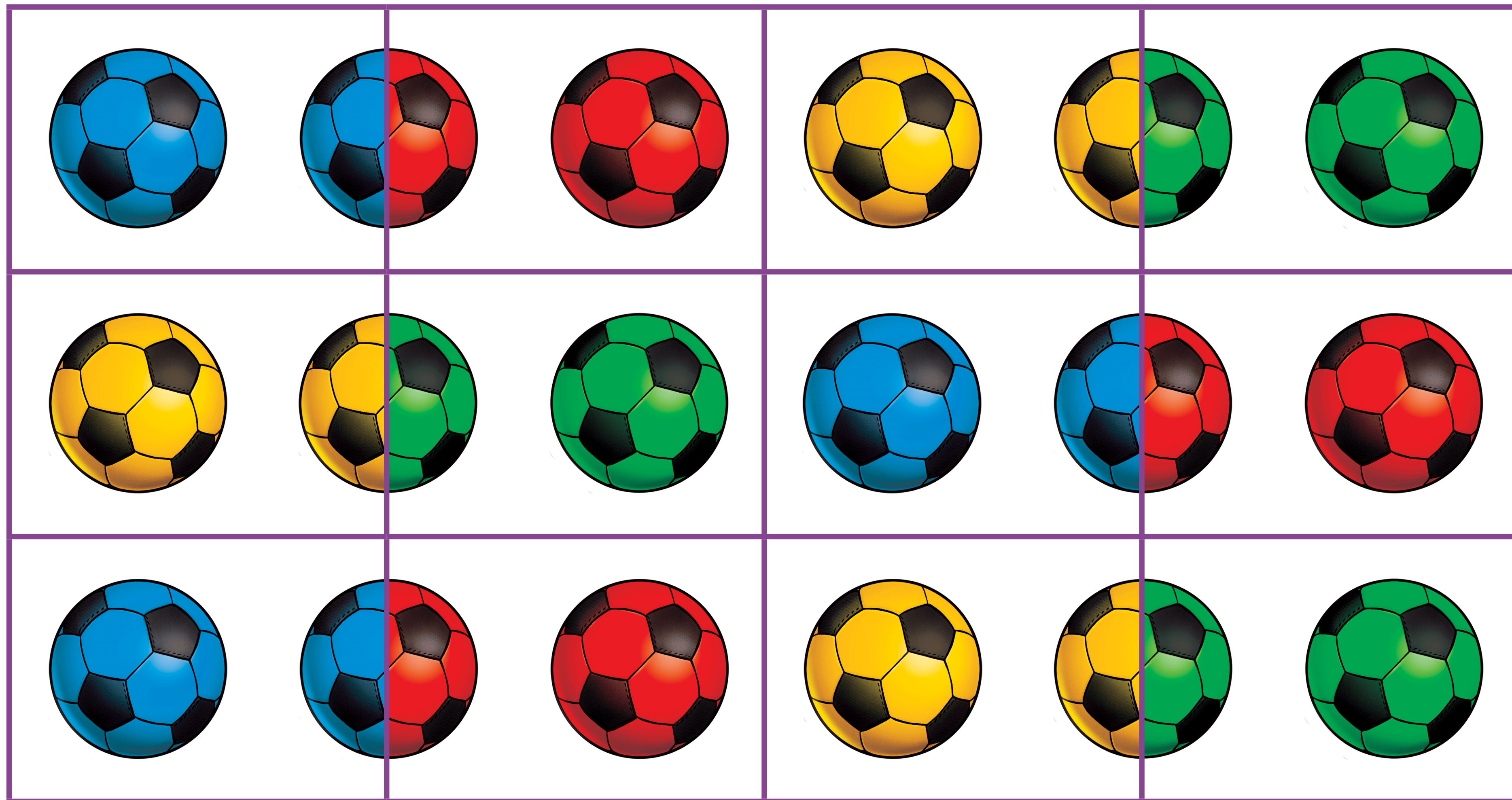
Ones

Remainder!

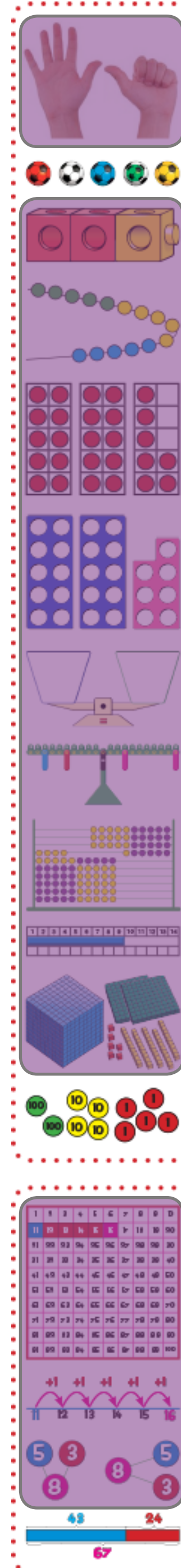
$065r4$
 $6 \overline{) 3^3 9^3 4}$



$$18 \div 1.5 = 12$$

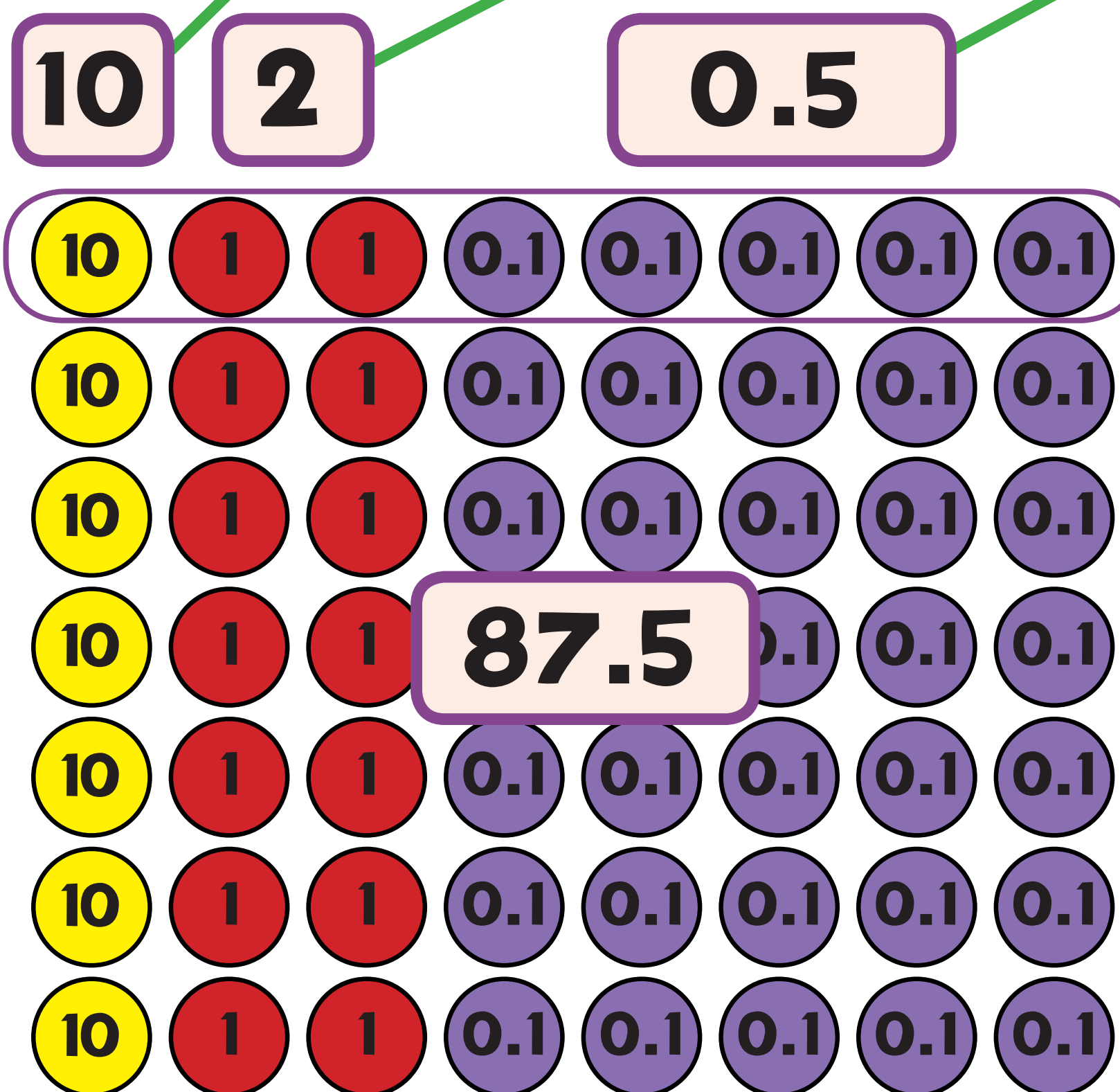
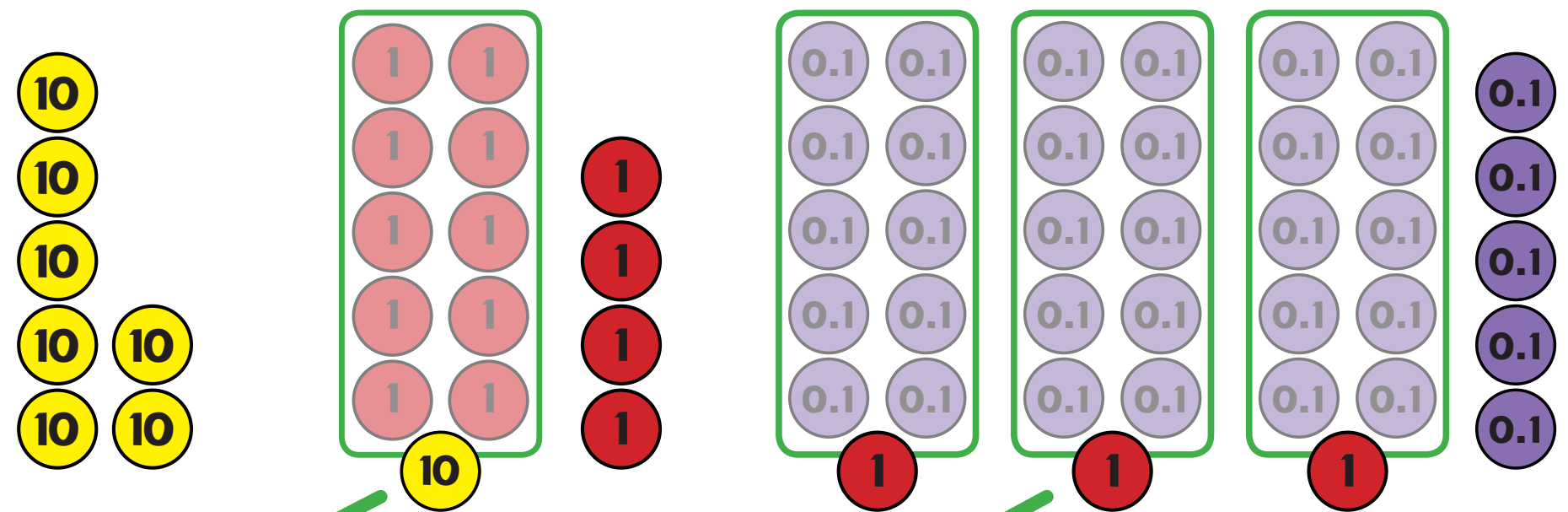


“How many 1.5s are there in 18?”



$$87.5 \div 7 = 12.5$$

$$\begin{array}{r} 70 \\ + 14 \\ + 3.5 \\ \hline 10 + 2 + 0.5 = 12.5 \end{array}$$



$$\begin{array}{r} 12.5 \\ 7 \overline{) 87.5} \\ \underline{7 } \\ 17 \\ \underline{14 } \\ 35 \\ \underline{35} \\ 0 \end{array}$$

